# INTRODUCTION TO STATISTICS

Slides by

**Pierre Dragicevic** 



## WHAT YOU WILL LEARN

**Statistical Applied** statistics theory This lecture

## GOALS

- Learn basic intuitions and terminology
- Perform basic statistical inference with R
- Focus on high-level aspects
- Accent on estimation rather than hypothesis testing ("the New Statistics")

## ORGANIZATION

- Part I Elementary notions
- Part II Tutorial with R
- Part III Assignments

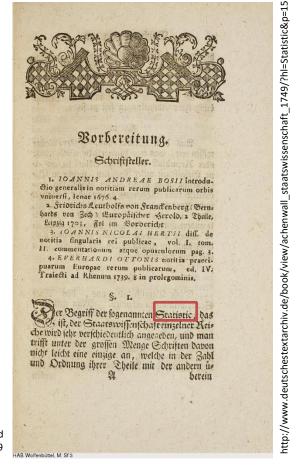
## A DEFINITION

Statistics is the study of the collection, analysis, interpretation, presentation and organization of data.

Dodge, Y. (2006) The Oxford Dictionary of Statistical Terms, OUP.

## ORIGINS

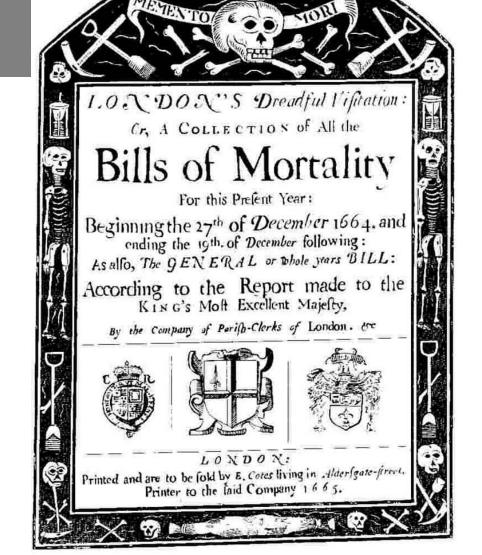
- 1750s German Statistik "analysis of data about the state"
- Quickly adopted in England (previously called "political arithmetics")



## ORIGINS

John Graunt, 1662 Observations on the bills of mortality





9613 384

21

102

537

66

101

21 21

99 392 356 213 269 191

80 134 105 76 73 130 282 62 105 260

59 80

Impostum e

Lethargy

Committee

livergrown, Spleen, and Rickets

Leproly

60 202 201 207 194 148 60

94 112

11

14

filled by feveral Accidents 28 54 25 16 20 26 ting's Evil 20 26 22 19

Cancer, Gangrene, and Fiftula	26	29	31	19	31	53	116	37	73	31	
Wolf Canker, Sore-mouth, and Thrush	66	28	54	42	68	51	5'3	72	44	81	
Childhed	161	The state of the s	114				158	192	177	201	ł
Chrisomes, and Infants	1369				1237	1280	1050	1243	1089	1393	1
Colick, and Wind	103		85	82	76			101	85		,
Cold, and Cough	News.		A second			work.	41	36	21	58	
Confumption, and Cough	2423	2200	2388	1938	2350	2410	2216	2868	2 606	3184	2
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Cut of the Stone	1	2	1	3		1	1	3	4		
Dropfy, and Tympany	185	434	421	508				704	660		
Drowned	47	40	30	27	49	50	13	30	43	45	1
Excellive drinking			2							22	1
Executed Park	8	17	29	43	24	13	19	21	19		1
Fainted in a Bath	1	100			1				1	2	
Falling-Sickness Flox, and fmall Pox	3			3		1 3	. 4	1		823	1
Found dead in the Streets	139	The state of the s	1190	184	525	1279	2		1294	7.73	
French-Pox	18	6	9	-0	7	3	14	4	3	23	
Frighted	10	29	15	18	21	20	10	20	29		
Gout	4	4	100		3		5	-	0	7	1
Grief	9	5	11			1		E .		12	
SHEL	12	13	16	7	17	14	11	17	10	1 16	

## ORIGINS

### John Graunt, 1662 Observations on the bills of mortality

- First "life tables"
- Dispelled several myths about the plague
- First analysis of sex ratio
- First realistic estimate of the population in London

## ORIGINS

- Prompted collection of more data
- Parallel developments in probability theory
- Statistics then developed into a more rigorous discipline and was applied to:
  - Business & industry
  - Medicine
  - Science

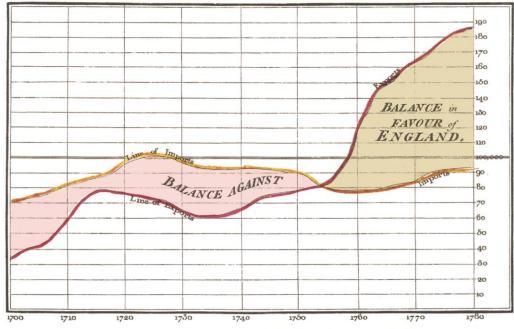
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## STATS & VISUALIZATION

#### **Statistical Charts**

William Playfair1759 - 1823





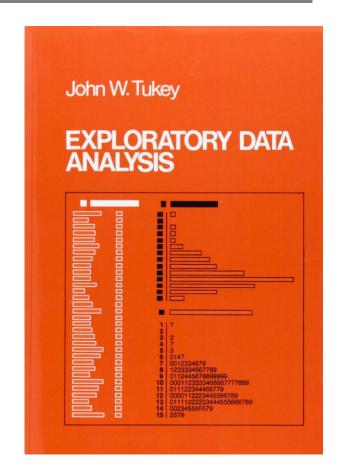
The Bottom line is divided into Years, the Right hand line into L10,000 each.

Published as the Act direct, 1st Major 1766, by W. Playtair

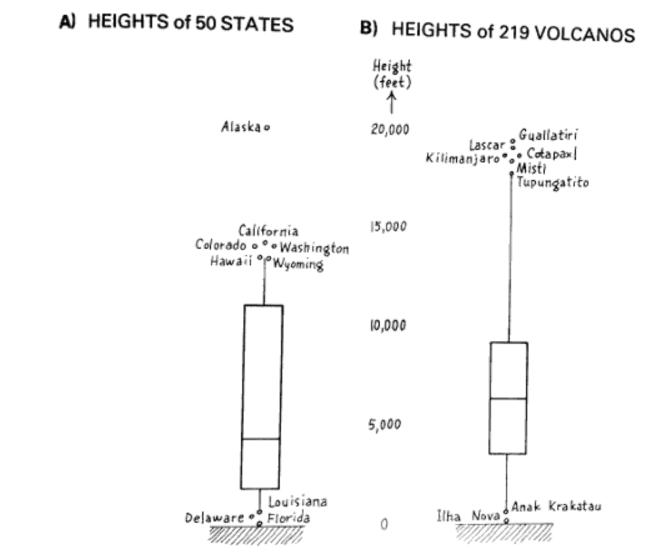
## STATS & VISUALIZATION

## **Exploratory Data Analysis**

- Tukey, 1977



#### Box-and-whisker plots with end values identified



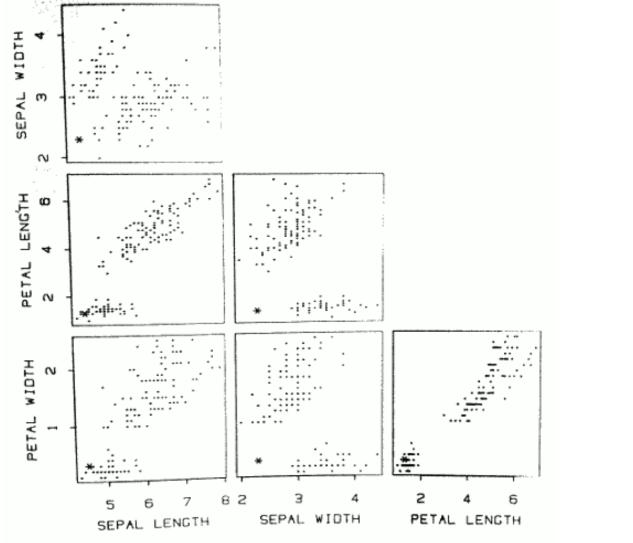
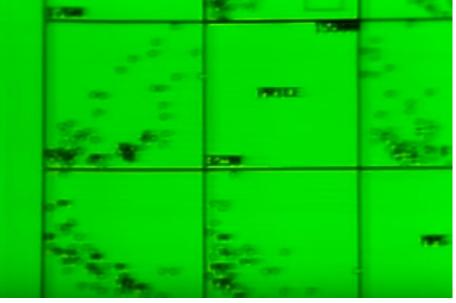


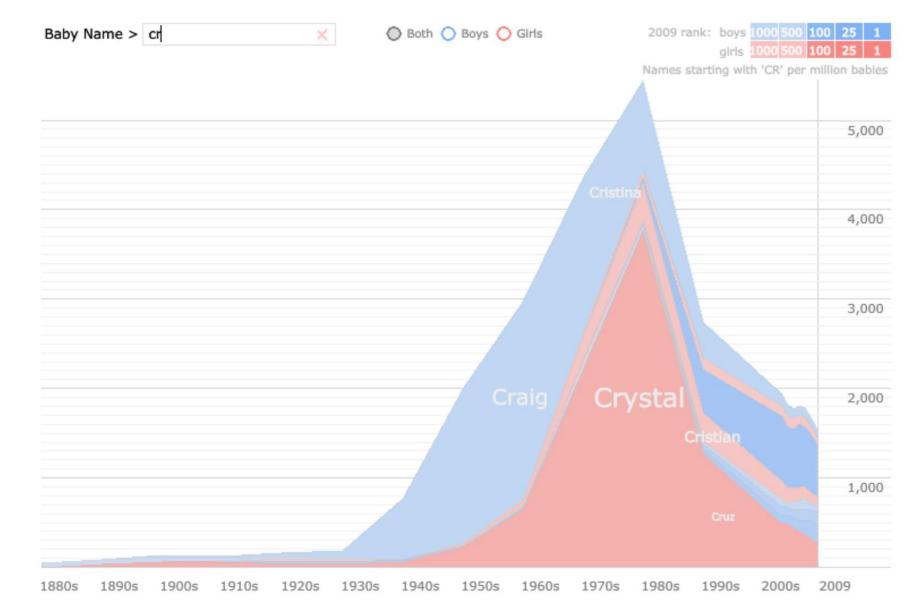
Figure 5.14 Generalized draftsman's display of the four-dimensional iris data (like Figure 5.11), with one flower plotted as an asterisk.

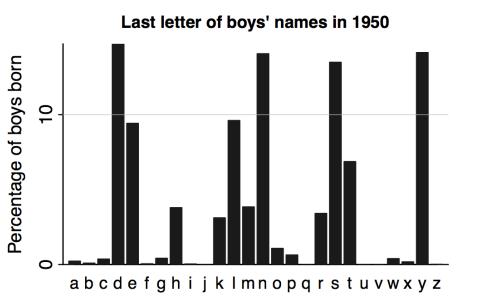
### **Statistical Graphics**

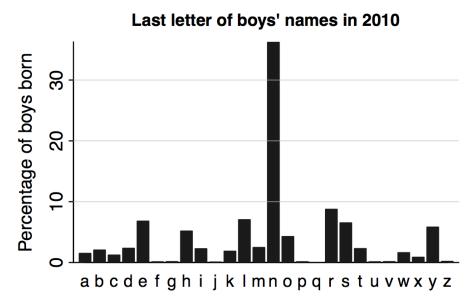
- AT&T Bell Labs Video, 1985











## 46 64 54 77 67 68 62 56 38 Population N = 9

Random

Sample 38 62 67 62 n = 4

$$\overline{X} = \frac{\sum X}{n} = \frac{229}{4} = 57.25$$

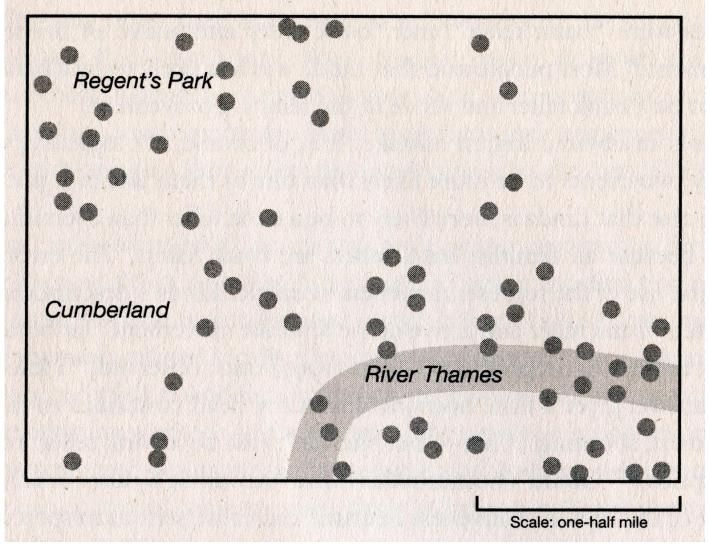
The mean of this Random Sample equals 57.25 (i.e.  $\overline{\chi} = 57.25$ )

 $\mu_{\times} = \frac{\sum \times}{N} = \frac{532}{9} = 59.11$ 

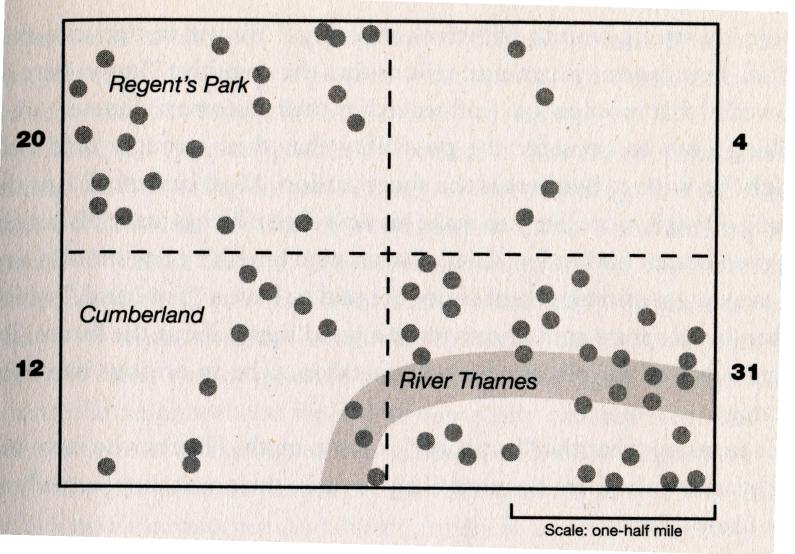
The Mean of this Population ( $\mu_{\times}$ ) equals 59.11 (i.e.  $\mu_{\times}$  = 59.11)

The Central Limit Theorem tells us that  $\overline{\chi}$  is an unbiased estimate of  $\mu_{ imes}$ . ( i.e.  $\overline{\overline{\chi}}$   $\longrightarrow$   $\mu_{ imes}$ )

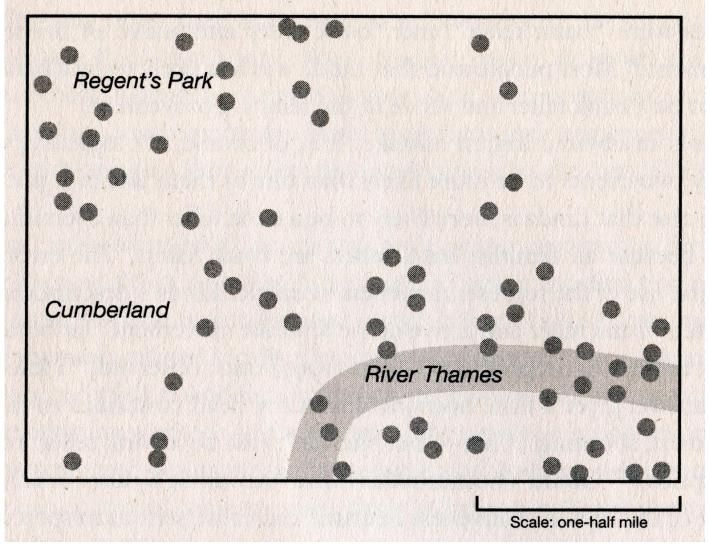
In short, with only one random sample to go on, the mean of the sample ( $\bar{\chi}$  = 57.25) is our best estimate of the population mean ( $\mu_{\chi}$ )



German bombings in London during WWII



German bombings in London during WWII

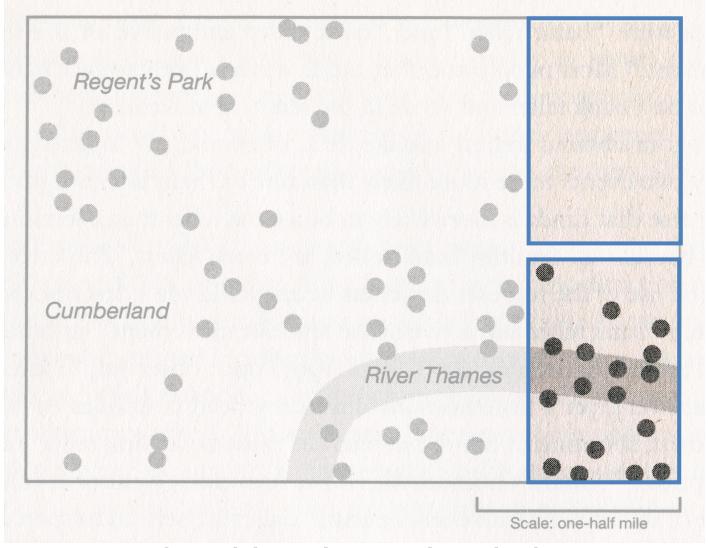


German bombings in London during WWII

## STATS & VISUALIZATION

#### Confirmatory data analysis

- For answering questions rigorously
- Example: is this new drug effective?
- Strong focus on automatic procedures, computation and objectivity
- Looking at data can impair objectivity:
  - Cherry picking, snooping, fishing, data mining



German bombings in London during WWII

## STATS & VISUALIZATION

Exploratory data analysis is sometimes compared to detective work: it is the process of gathering evidence.

Confirmatory data analysis is comparable to a court trial: it is the process of evaluating evidence.

Exploratory analysis and confirmatory analysis "can—and should—proceed side by side" (Tukey; 1977).

## WHAT ARE STATS?

- A set of tools and methods
- With an old tradition:
  - Origins in demographics
  - Anchored in mathematics & probability theory
  - Visual representations play a role
  - A generally strong focus on (computationally cheap) numerical calculations

## WHAT ARE STATS?

#### Good for:

- Summarizing data for presentation
- Answering questions rigorously
- Making predictions
- Making rational, evidence-based decisions
- A long accumulated experience!

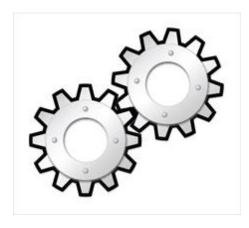
# STATISTICAL TOOLS



## STATISTICAL TOOLS

**DESCRIPTIVE STATISTICS** 

INFERENTIAL STATISTICS



# STATISTICAL TOOLS

#### **DESCRIPTIVE STATISTICS**



## AN EXAMPLE

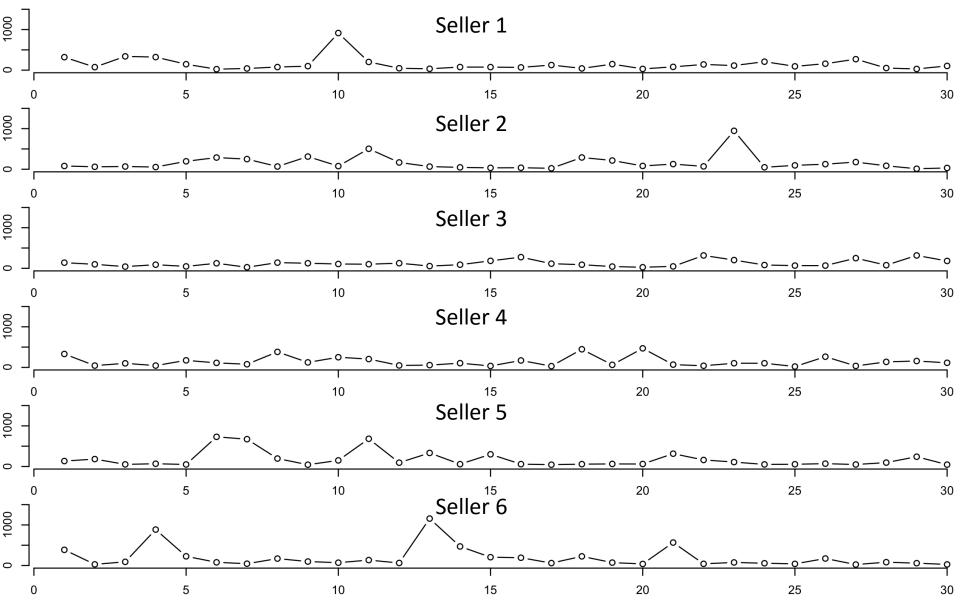
Selling encyclopedias

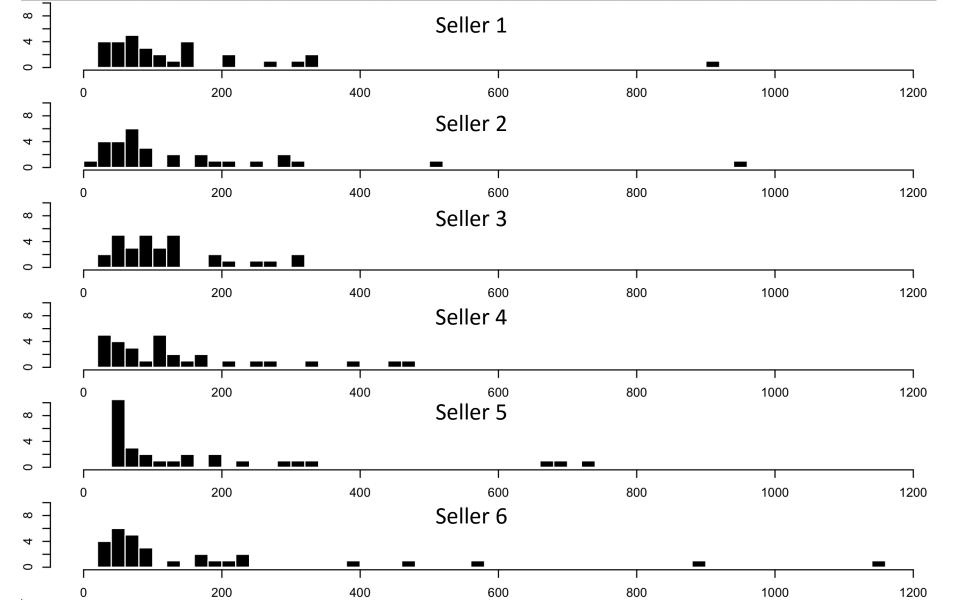




day	Seller 1	Seller 2	Seller 3	Seller 4	Seller 5	Seller 6
1	€320	€80	€139	€330	€133	€387
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6	€24	€288	€124	€111	€730	€79
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8	€76	€67	€140	€382	€195	€171
9	€99	€312	€125	€123	€43	€98
10	€915	€77	€106	€250	€149	€70
11	€202	€504	€101	€205	€682	€134
12	€47	€167	€126	€48	€93	€63
13	€34	€65	€55	€56	€333	€1,157
14	€76	€46	€89	€104	€56	€470
15	€75	€34	€184	€35	€299	€205
16	€68	€37	€275	€170	€57	€192

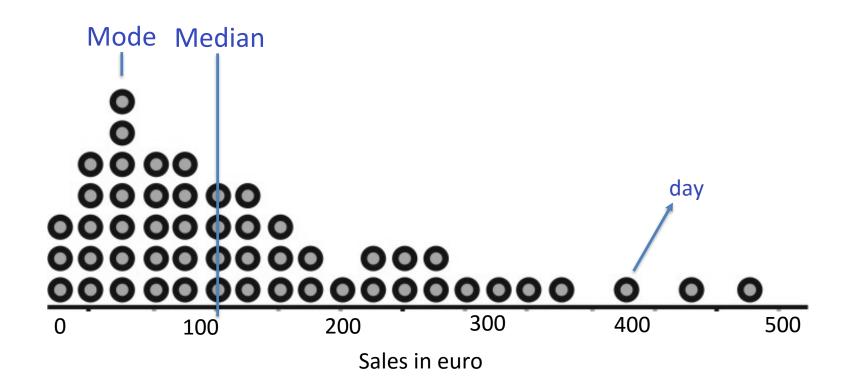
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15	€75	€34	€184	€35	€299	€205
16	€68	€37	€275	€170	€57	€192
17	€126	€23	€114	€30	€43	€60
18	€43	€290	€89	€446	€57	€226
19	€149	€215	€43	€63	€62	€72
20	€31	€81	€26	€469	€60	€39
21	€81	€127	€47	€68	€315	€566
22	€141	€70	€317	€40	€160	€42
23	€113	€947	€203	€102	€108	€76
24	€209	€48	€81	€102	€50	€56
25	€94	€95	€67	€21	€54	€41
26	€159	€125	€67	€263	€69	€173
27	€271	€176	€250	€35	€48	€24
28	€52	€85	€77	€136	€95	€82
29	€30	€12	€317	€157	€240	€58
30	€104	€31	€181	€113	€45	€27





# CENTRAL TENDENCY

Name & Meaning	Formula / Example	Used for			
Arithmetic Mean [average]	$\frac{sum}{size} = \frac{a+b+c}{3}$	Most situations ("average item")			
Median [middle value]	Middle of sorted list (2 middles? Average 'em)	Wildly varying samples (houses, incomes)			
Mode [most popular]	Most popular value	No compromises (winner takes all)			
Geometric Mean [average factor]	$\sqrt[3]{abc}$	Investments, growth, area, volume			
Harmonic Mean [average rate]	$\frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$	Speed, production, cost			

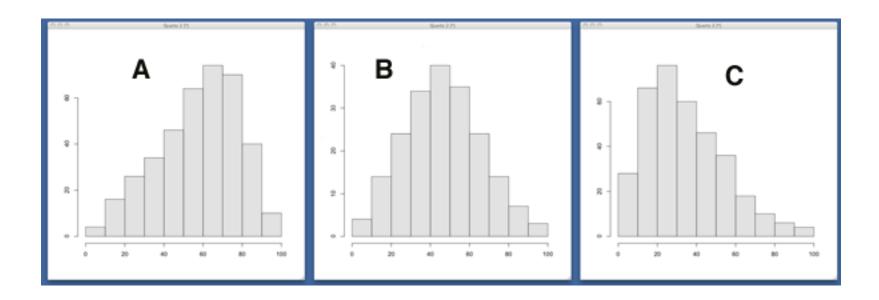


When are the mean and the median equal? When do they differ?

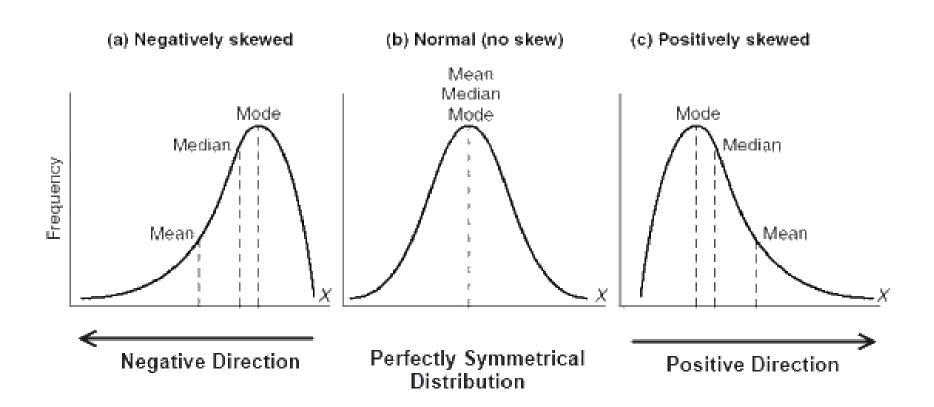
negative skew

symmetric

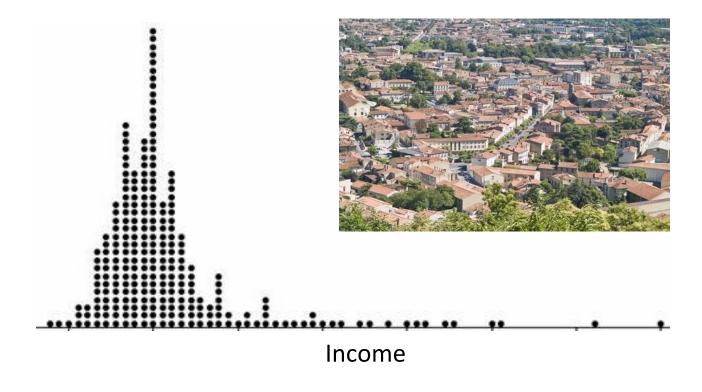
positive skew







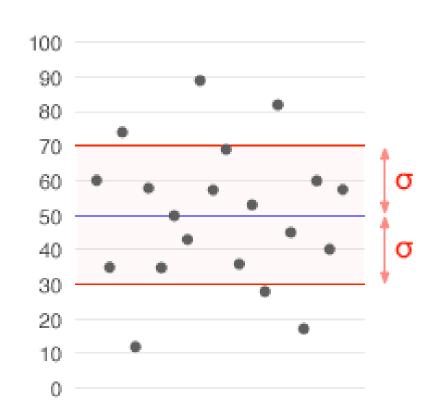
What is the best measure of central tendency?



# DISPERSION

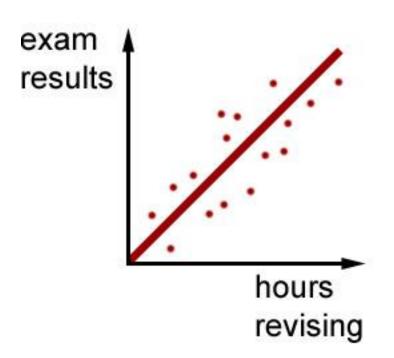
### **Standard Deviation**

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}$$



# DEPENDENCE

#### Correlation

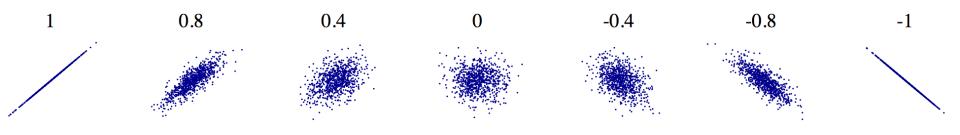


POSITIVE CORRELATION

 people who do more revision get higher exam results.

# DEPENDENCE

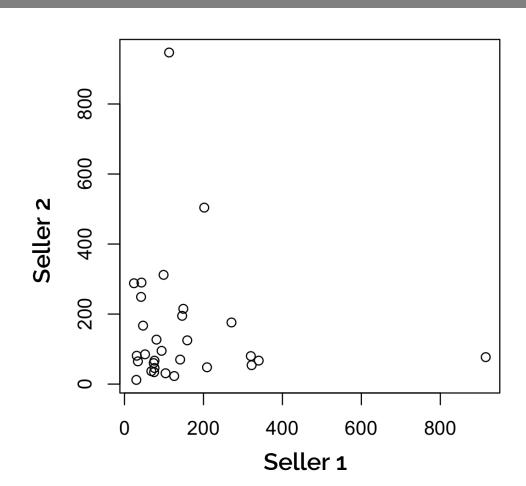
#### Correlation

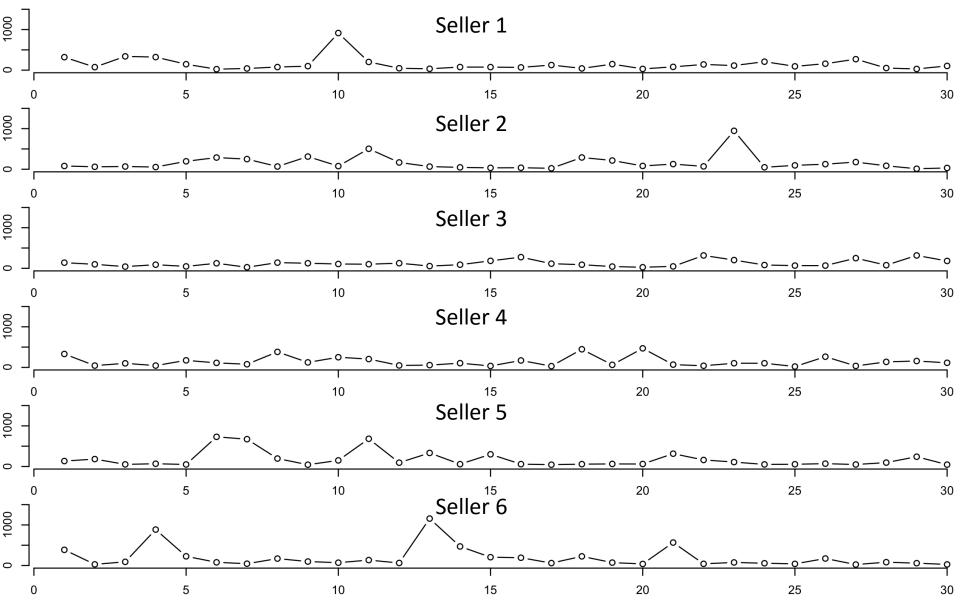


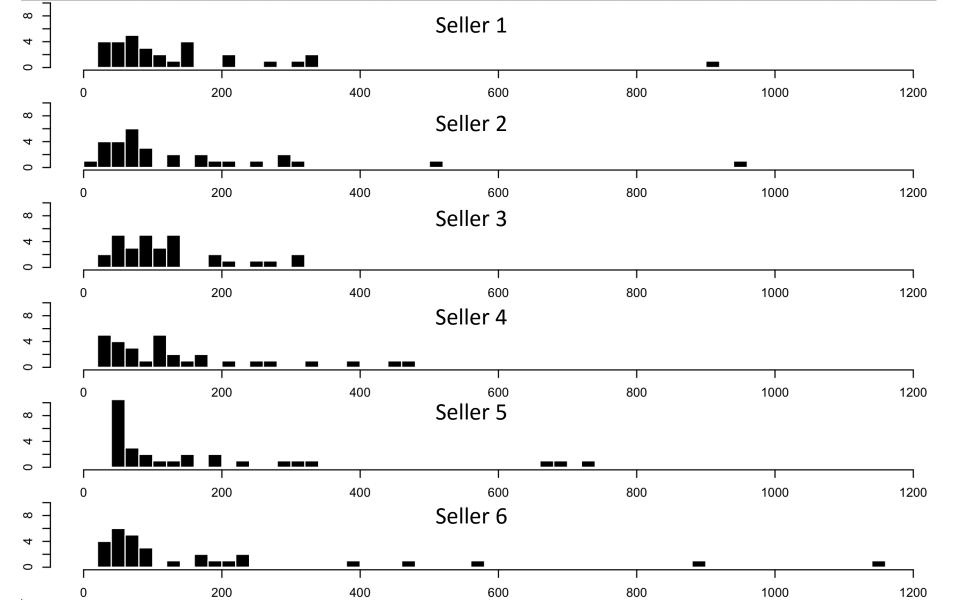
## DEPENDENCE

### Correlation

r = -0.08



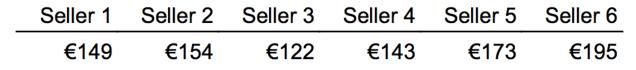


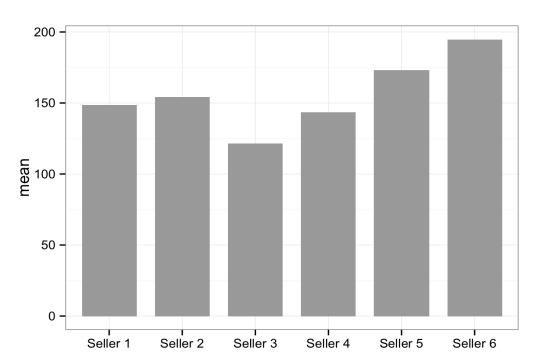


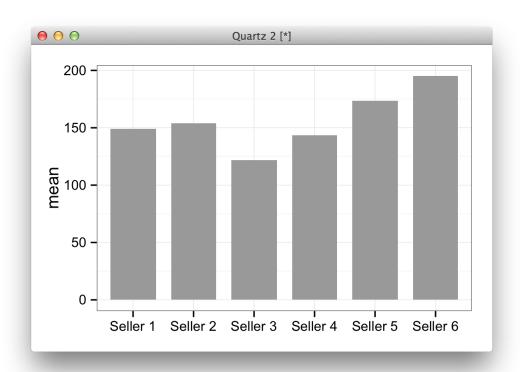
#### **Average Sales**

Seller 1	Seller 2	Seller 3	Seller 4	Seller 5	Seller 6
€149	€154	€122	€143	€173	€195

#### **Average Sales**

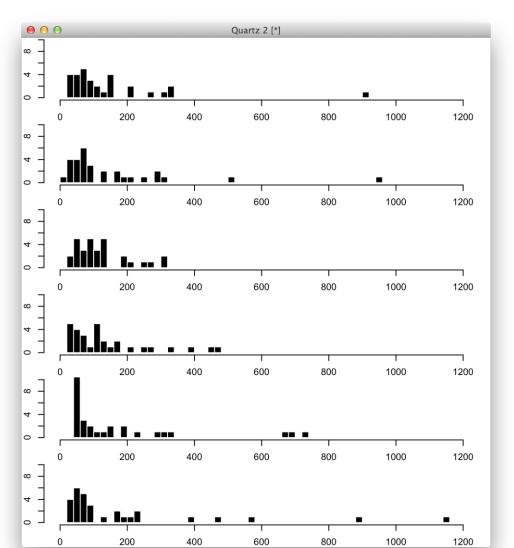






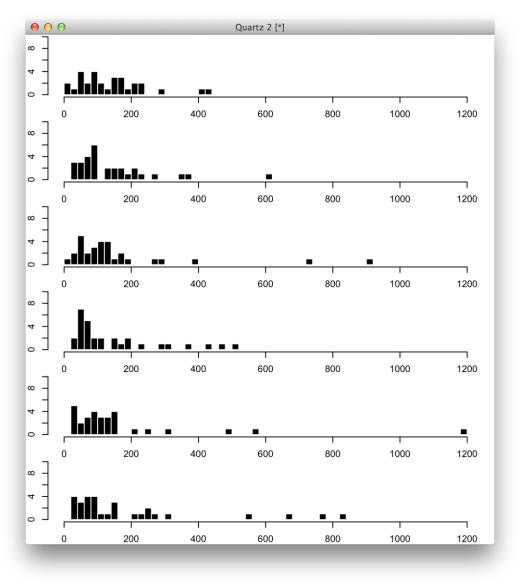
### How much can we trust this chart?

## LET US TRAVEL TO THE FUTURE

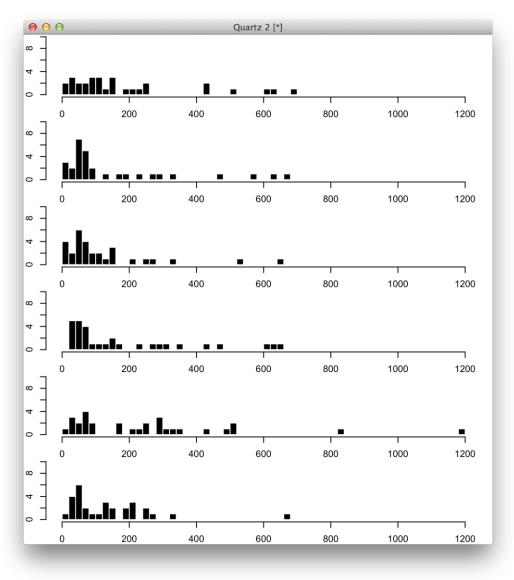


#### September 2016

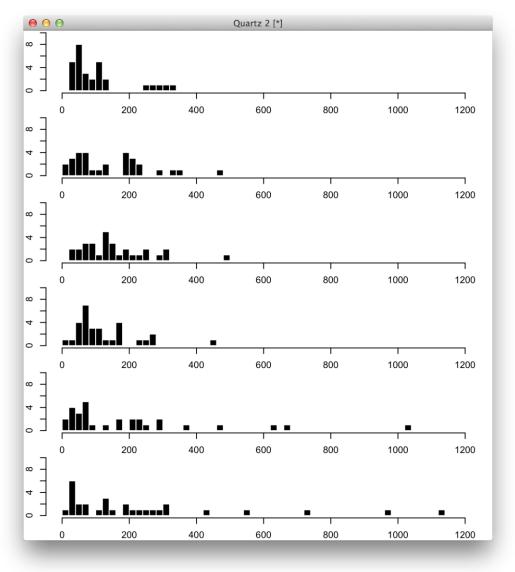
#### October 2016



#### November 2016







#### September 2016



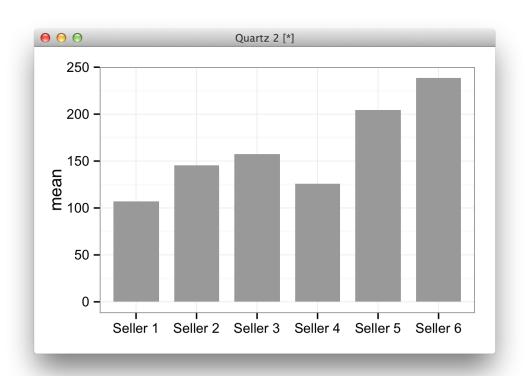
#### October 2016



#### November 2016



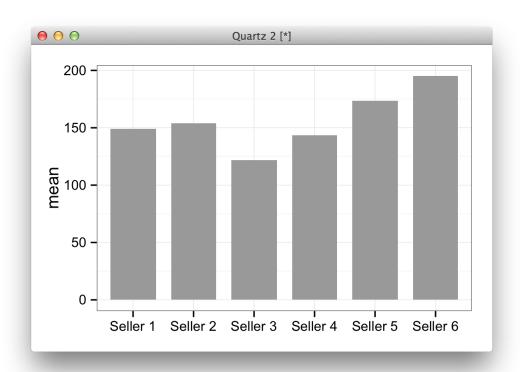
#### December 2016



# BACK TO THE PRESENT

#### September 2016

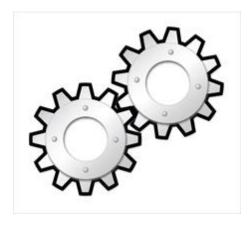
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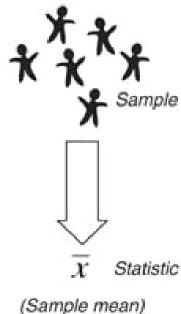


### How much can we trust this chart?

# STATISTICAL TOOLS

### INFERENTIAL STATISTICS





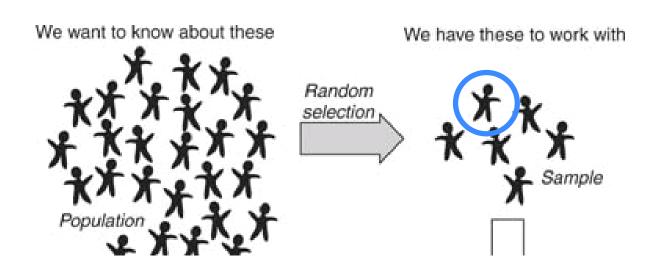
We want to know about these We have these to work with Random selection Sample Population Inference Parameter Statistic (Population mean) (Sample mean)

## Terminology:

### Sample vs. population

- Mean, median, standard deviation, correlation, etc.
  - A sample statistic
  - A population parameter

### Unit of statistical analysis



= "the thing that I'm sampling from a larger population"

## Unit of statistical analysis

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### Unit of statistical analysis

day	Seller 1		
1	€320		
2	€74		
3	€340		
4	€322		
5	€146		
6	€24		
7	€42		
8	€76		
9	€99		
10	€915		

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# STATISTICAL INFERENCE

### Unit of statistical analysis

#### **Average Sales**

_	Seller 1	Seller 2	Seller 3	Seller 4	Seller 5	Seller 6
	€149	€154	€122	€143	€173	€195

# STATISTICAL INFERENCE

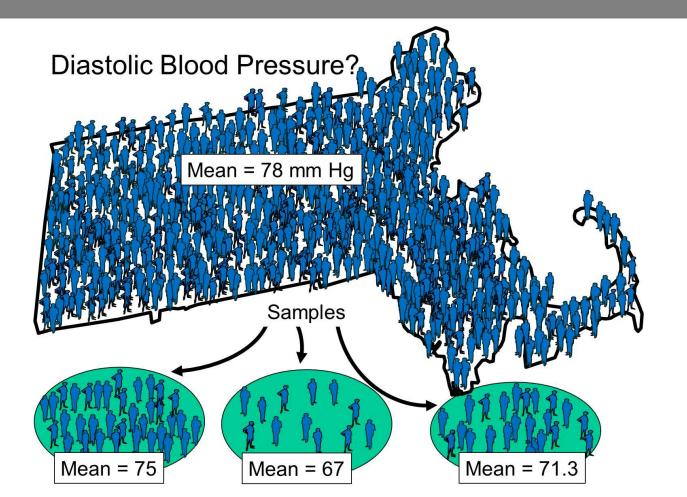
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"The sampling distribution of a statistic is the distribution of that statistic, considered as a random variable, when derived from a random sample of size n."

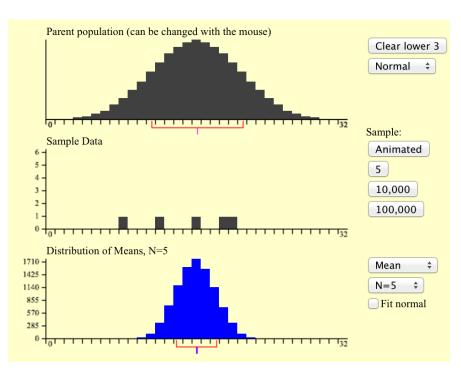
[...]

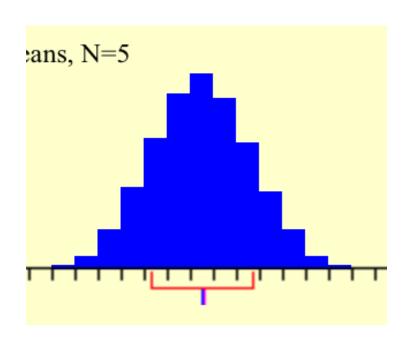
"It may be considered as the distribution of the statistic for all possible samples from the same population of a given size"

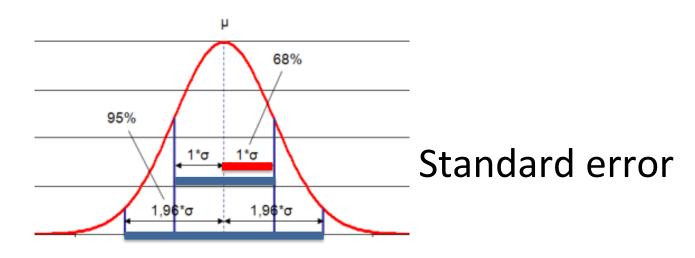


Demo

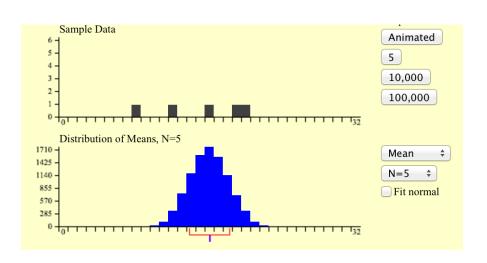
http://onlinestatbook.com/stat\_sim/sampling\_dist/



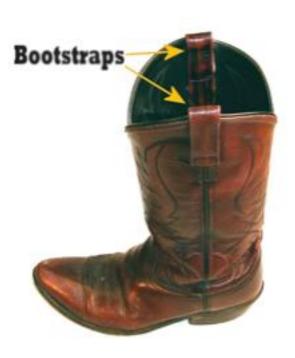




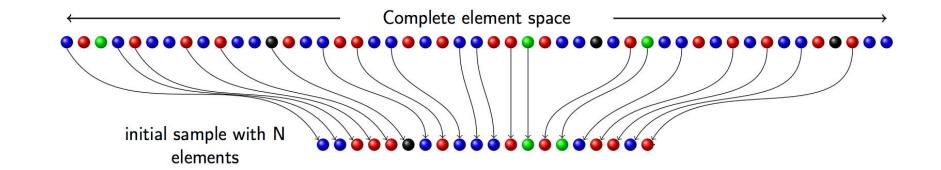
95% confidence interval

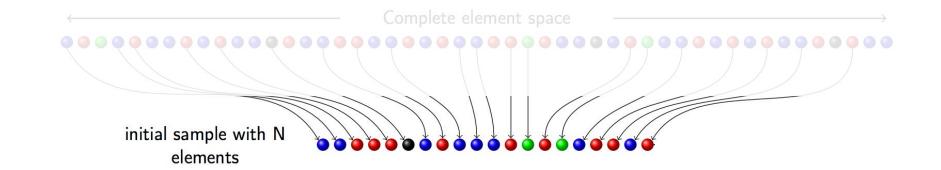


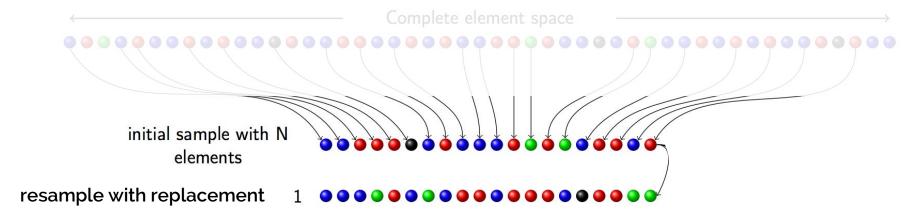
- Resampling techniques
  - Bootstrapping

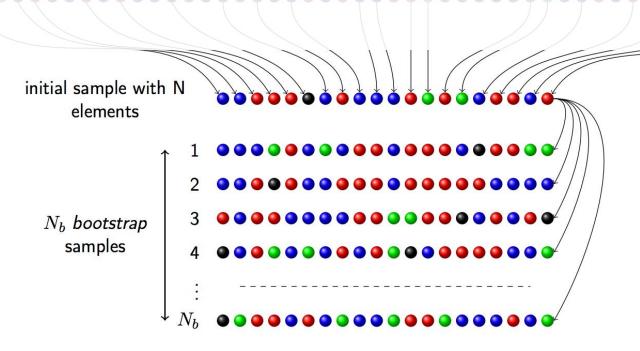


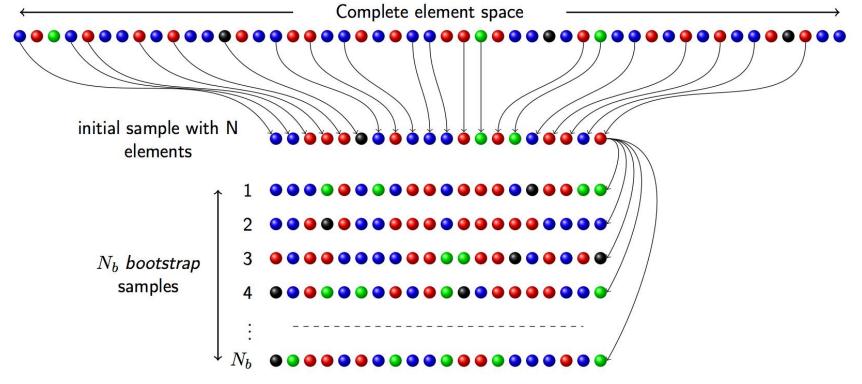








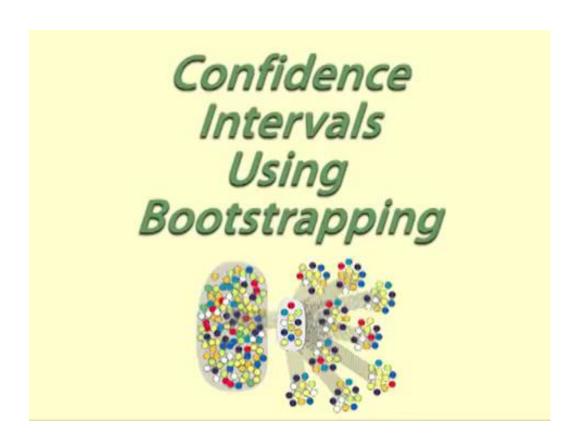




#### Theorem (B. Efron, Ann. Statist. 1979)

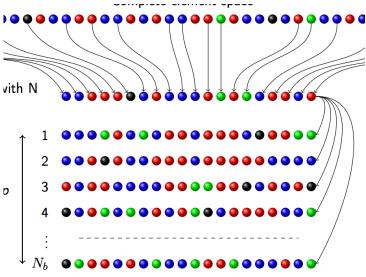
When N tend to infinity, the distribution of average values computed from bootstrap samples is equal to the distribution of average values obtained from ALL samples with N elements which can be constructed from the complete space. Thus the width of the distribution gives an evaluation of the sample quality.

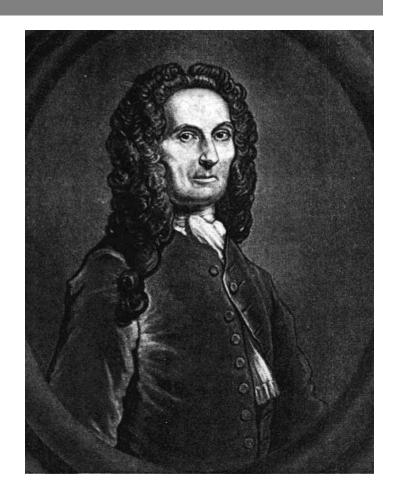
 Bootstrapping video



 How did people this do before computers?







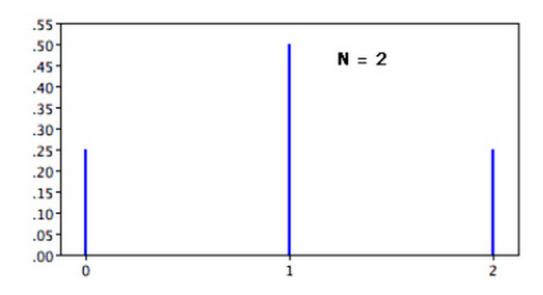


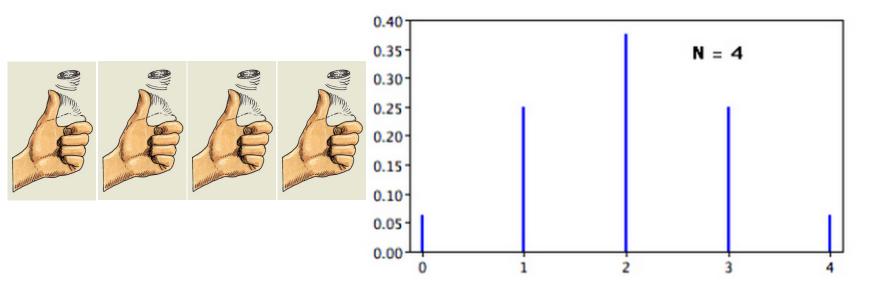


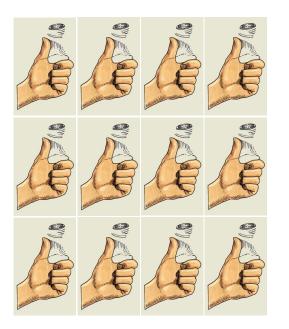


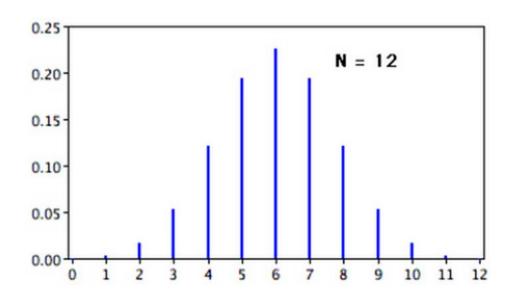




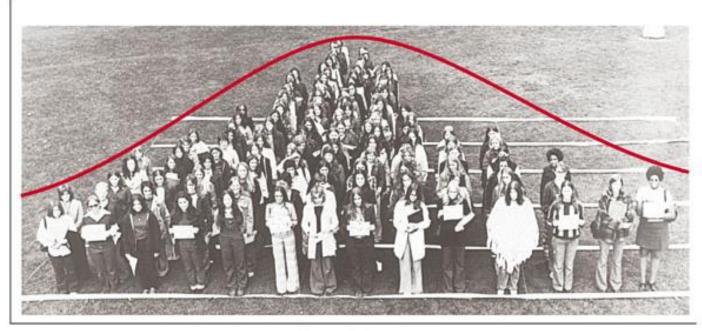




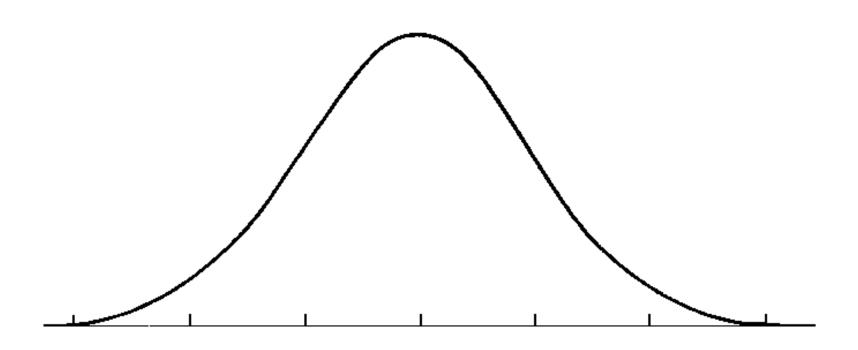




Number of individuals

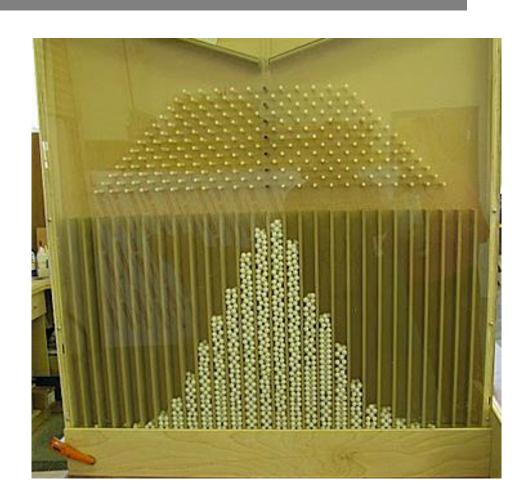


Height in inches



Sir Francis Galton
 1822 – 1911

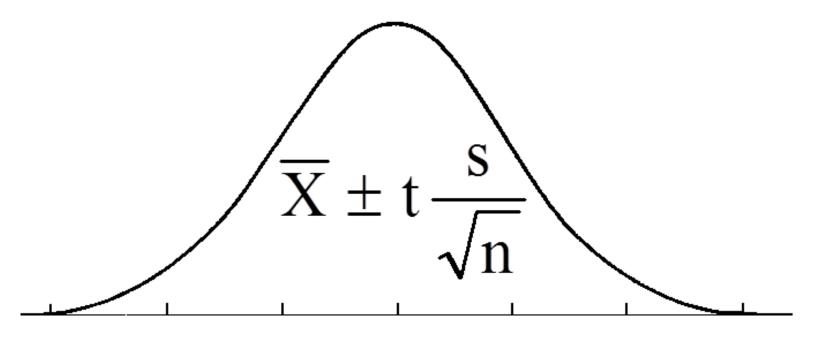
Bean Machine or Galton Board:



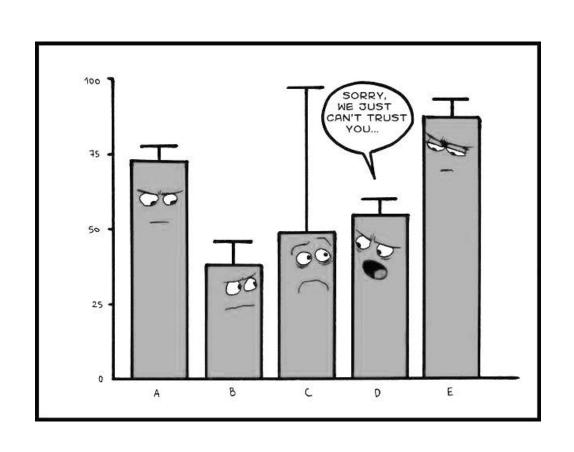
#### **Central Limit Theorem**

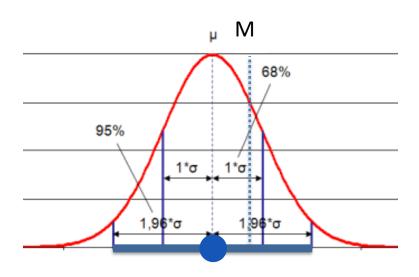
Given certain conditions, the arithmetic mean of a sufficiently large number of iterates of independent random variables, each with a well-defined expected value and well-defined variance, will be approximately normally distributed

"Exact" Confidence Intervals

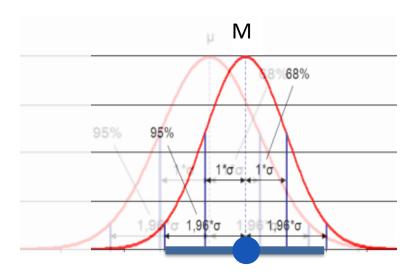


t ~ 1.96 for large samples

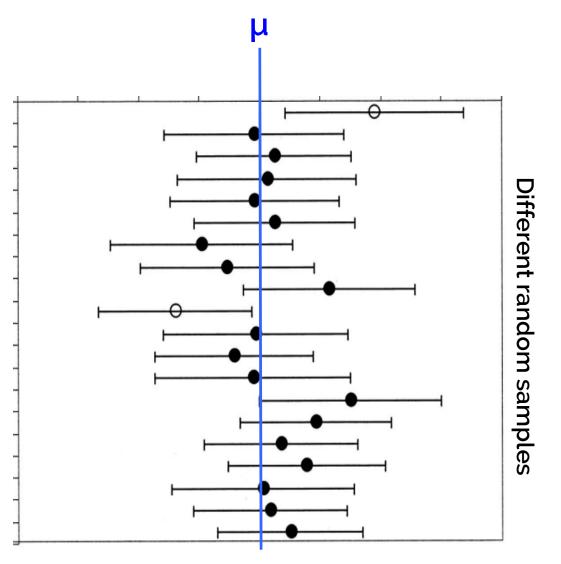




margin of error = length of blue line



95% confidence interval



tinyurl.com/danceptrial2

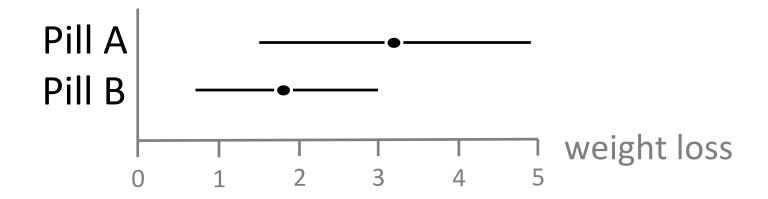
- Several interpretations
- « a range of plausible values for μ. Values outside the Cl are relatively implausible. » (Cumming and Finch, 2005)
- Examples of presentation formats:

```
2.2m, 95% CI [1.6m, 2.8m]
```

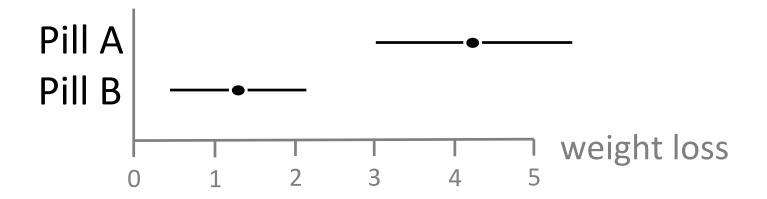
from 1.6m to 2.8m



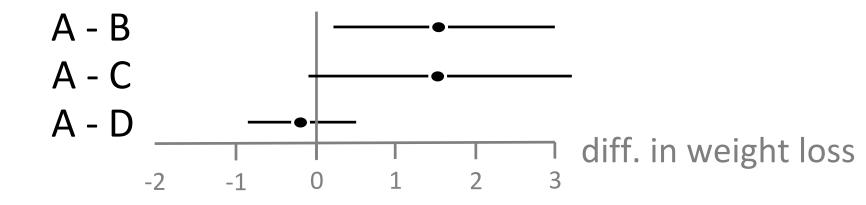
« a range of plausible values for  $\mu$ . Values outside the CI are relatively implausible. » (Cumming and Finch, 2005)



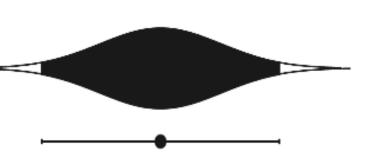
« a range of plausible values for μ. Values outside the CI are relatively implausible. » (Cumming and Finch, 2005)



 « a range of plausible values for μ. Values outside the Cl are relatively implausible. » (Cumming and Finch, 2005)

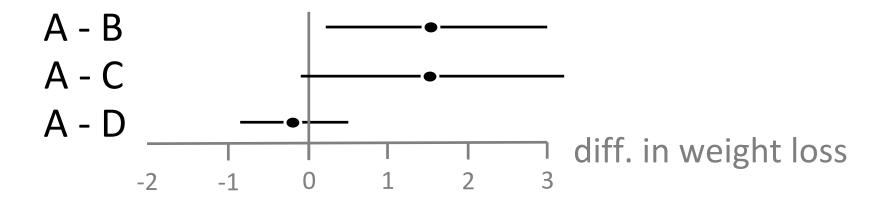


"values close to our M are the best bet for  $\mu$ , and values closer to the limits of our CI are successively less good bets."



(Cumming, 2013)

# BE VAGUE IN YOUR DESCRIPTION

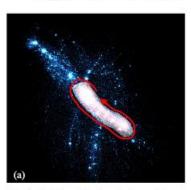


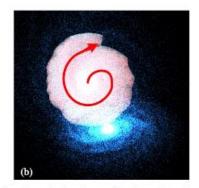
"the figure provides good evidence that B outperforms A, whereas C and A seem very similar, and results are largely inconclusive concerning the difference between D and A."

### PUBLISHED EXAMPLE

#### CAST: Effective and Efficient User Interaction for Context-Aware Selection in 3D Particle Clouds

Lingyun Yu, Konstantinos Efstathiou, Petra Isenberg, and Tobias Isenberg, Senior Member, IEEE





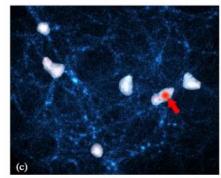


Fig. 1. (a) SpaceCast selects particle clusters by enclosing them with a lasso, based on the lasso shape; (b) TraceCast does not require an accurate lasso; and (c) with PointCast users can select tiny clusters from a noisy environment with only a single click or touch.

Abstract—We present a family of three interactive Context-Aware Selection Techniques (CAST) for the analysis of large 3D particle datasets. For these datasets, spatial selection is an essential prerequisite to many other analysis tasks. Traditionally, such interactive target selection has been particularly challenging when the data subsets of interest were implicitly defined in the form of complicated structures of thousands of particles. Our new techniques SpaceCast, TraceCast, and PointCast improve usability and speed of spatial selection in point clouds through novel context-aware algorithms. They are able to infer a user's subtle selection intention from gestural input, can deal with complex situations such as partially occluded point clusters or multiple cluster layers, and can all be fine-tuned after the selection interaction has been completed. Together, they provide an effective and efficient tool set for the fast exploratory analysis of large datasets. In addition to presenting Cast, we report on a formal user study that compares our new techniques not only

### PUBLISHED EXAMPLE

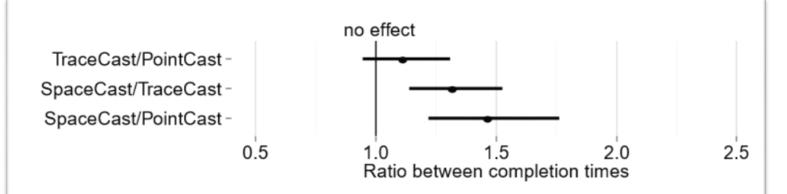


Fig. 6. Ratios between mean completion times for the Cast selection techniques. Error bars show 95% confidence intervals.

Thus, overall we have good evidence that both Point-Cast and TraceCast outperform SpaceCast, and some indication that PointCast may outperform TraceCast. At any rate, the differences among Cast methods are marginal compared to the differences between each Cast method and CloudLasso or CylinderSelection.

# BACK TO OUR EXAMPLE

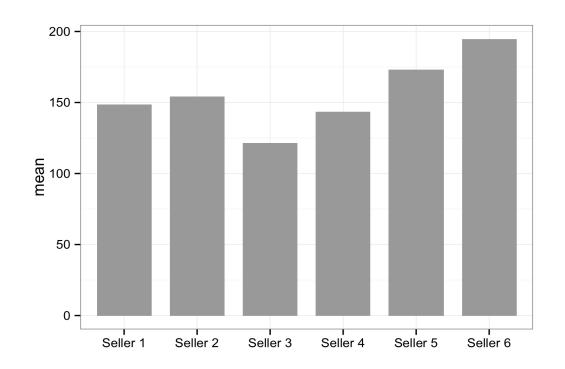
Selling encyclopedias





#### **Average Sales**

 Seller 1	Seller 2	Seller 3	Seller 4	Seller 5	Seller 6
€149	€154	€122	€143	€173	€195



#### Fair Statistical Communication in HCI

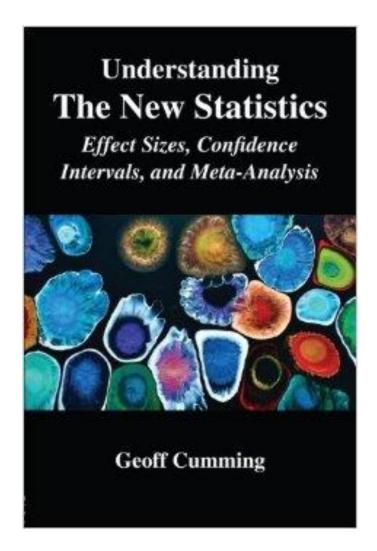
Pierre Dragicevic

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Preprint v1.6.3, to appear in February 2016. How to discussion of the desired property of the second property of t
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**Abstract** Statistics are tools to help end users accomplish their task. In research, to be qualified as usable, statistical tools should help researchers advance scientific knowledge by supporting and promoting the effective communication of research findings. Yet areas such as human-computer interaction (HCI) have adopted tools—i.e., *p*-values and dichotomous testing procedures—that have proven to be poor at supporting these tasks. The abusive use of these procedures has been severely criticized in a range of disciplines for several decades, suggesting that tools should be blamed, not end users. This chapter explains in a non-technical manner why it would be beneficial for HCI to switch to an *estimation* approach, i.e., reporting informative charts with effect sizes and interval estimates, and offering nuanced interpretations of our results. Advice is offered on how to communicate our empirical results in a clear, accurate, and transparent way without using any tests or *p*-values.

#### 1 Introduction

A common analogy for statistics is the toolbox. As it turns out, researchers in human-computer interaction (HCI) study computer tools. A fairly uncontroversial position among them is that tools should be targeted at end users, and that we should



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CONTAC

#### Bad Stats: Not What It Seems

#### Towards a Statistical Reform in HCI and Visualization

Pierre Dragicevic and colleagues



This page provides arguments and reading material to explain why it would be beneficial for humancomputer interaction (HCI) and information visualization (infovis) to stop doing mindless null hypothesis significance testing (NHST) and start reporting informative charts with effect sizes and interval estimates, as well as offering more nuanced interpretations of our results. Our scientific standards can also be greatly improved by planning analyses and sharing experimental material online.

#### Content:

Fair Statistical Communication in HCI (book chapter)

Bad Stats are Miscommunicated Stats (BELIV 2014 Keynote)

Running an HCI Experiment in Multiple Parallel Universes (Alt.CHI 2014 Paper)

Quotes about null hypothesis significance testing (NHST)

Links

Reading List

More Readings

Papers (somehow) in favor of NHST

Papers against confidence intervals

Papers from the HCI Community



http://tinyurl.com/stats-dresden