

# *Information Visualization*

Multi-Dimensional Data Visualization

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*after today you will...*

- have learned techniques for MD data visualization
- have learned relationships between MD visualization techniques
- have understood the problems of MD data visualization

***WHAT IS MULTIDIMENSIONAL DATA?***

# *Data sets*

- Data comes in many different forms
- Typically, not in the way you want it
- **Heterogeneous** data often seen as multiple dimensions of elements extracted by *different* patterns or needs.

# A data set!

164	dodge-st.-reg	16.0	8.	400.0	170.0	4668.	11.5	75.	1.
165	fiat-124-spor	15.0	8.	350.0	145.0	4440.	14.0	75.	1.
166	fiat-124-tc	16.0	8.	318.0	150.0	4498.	14.5	75.	1.
167	fiat-124b	14.0	8.	351.0	148.0	4657.	13.5	75.	1.
168	fiat-128	17.0	6.	231.0	110.0	3907.	21.0	75.	1.
169	fiat-128	16.0	6.	250.0	105.0	3897.	18.5	75.	1.
170	fiat-131	15.0	6.	258.0	110.0	3730.	19.0	75.	1.
171	fiat-strada-cu	18.0	6.	225.0	95.00	3785.	19.0	75.	1.
172	fiat-x1.9	21.0	6.	231.0	110.0	3039.	15.0	75.	1.
173	ford-country	20.0	8.	262.0	110.0	3221.	13.5	75.	1.
174	ford-country	13.0	8.	302.0	129.0	3169.	12.0	75.	1.
175	ford-country	29.0	4.	97.00	75.00	2171.	16.0	75.	3.
176	ford-escort-2	23.0	4.	140.0	83.00	2639.	17.0	75.	1.
177	ford-escort-4	20.0	6.	232.0	100.0	2914.	16.0	75.	1.
178	ford-f108	23.0	4.	140.0	78.00	2592.	18.5	75.	1.
179	ford-f250	24.0	4.	134.0	96.00	2702.	13.5	75.	3.
180	ford-fairmon	25.0	4.	90.00	71.00	2223.	16.5	75.	2.
181	ford-fairmon	24.0	4.	119.0	97.00	2545.	17.0	75.	3.
182	ford-fairmon	18.0	6.	171.0	97.00	2984.	14.5	75.	1.

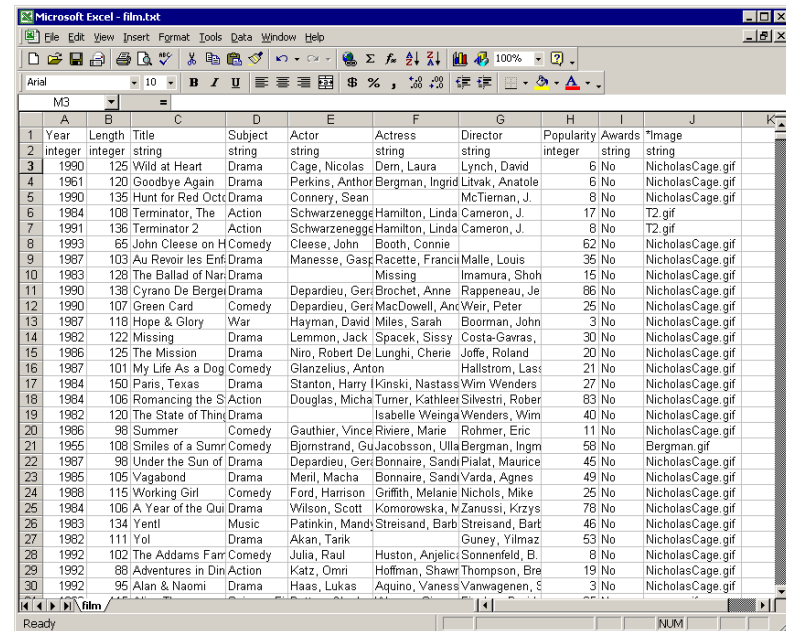
# Schema

- Cars
  - brand
  - model
  - year
  - cost
  - size
  - weights
  - miles per gallon



# Data Tables

- Often, we take raw data and transform it into a form that is more workable
- Main idea:
  - Individual items are called cases (rows)
  - Cases have **variables** (columns)



The screenshot shows a Microsoft Excel spreadsheet with the following data:

Year	Length	Title	Subject	Actor	Actress	Director	Popularity	Awards	Image
1990	125	Wild at Heart	Drama	Cage, Nicolas	Dem, Laura	Lynch, David	6	No	NicholasCage.gif
1961	120	Goodbye Again	Drama	Perkins, Anthor	Bergman, Ingrid	Ltvak, Anatole	6	No	NicholasCage.gif
1990	135	Hunt for Red Oct	Drama	Connelly, Sean	McTiernan, J.		8	No	NicholasCage.gif
1984	108	Terminator, The	Action	Schwarzenegger	Hamilton, Linda	Cameron, J.	17	No	T2.gif
1991	136	Terminator 2	Action	Schwarzenegger	Hamilton, Linda	Cameron, J.	8	No	T2.gif
1993	65	John Cleese on H	Comedy	Cleese, John	Booth, Connie		62	No	NicholasCage.gif
1987	103	Au Revoir les Enf	Drama	Manesse, Gas	Racette, Francis	Malle, Louis	35	No	NicholasCage.gif
1983	128	The Ballad of Nar	Drama		Missing	Imamura, Sho	15	No	NicholasCage.gif
1990	138	Cyrano De Berge	Drama	Depardieu, Geri	Brochet, Anne	Rappeneau, Je	86	No	NicholasCage.gif
1990	107	Green Card	Comedy	Depardieu, Geri	MacDowell, Anc	Weir, Peter	25	No	NicholasCage.gif
1987	118	Hope & Glory	War	Hayman, David	Miles, Sarah	Boorman, John	3	No	NicholasCage.gif
1982	122	Missing	Drama	Lemmon, Jack	Spacek, Sissy	Costa-Gavras,	30	No	NicholasCage.gif
1986	125	The Mission	Drama	Niro, Robert De	Lunghi, Cherie	Joffe, Roland	20	No	NicholasCage.gif
1987	101	My Life As a Dog	Comedy	Glanzelius, Anton		Hallstrom, Lass	21	No	NicholasCage.gif
1984	150	Paris, Texas	Drama	Stanton, Harry	IKnski, Nastass	Wim Wenders	27	No	NicholasCage.gif
1984	106	Romancing the S	Action	Douglas, Micha	Turner, Kathleen	Silvestri, Rober	83	No	NicholasCage.gif
1982	120	The State of Thing	Drama		Isabelle Weinga	Wenders, Wim	40	No	NicholasCage.gif
1986	96	Summer	Comedy	Gauthier, Vince	Riviere, Marie	Rohmer, Eric	11	No	NicholasCage.gif
1955	108	Smiles of a Sumr	Comedy	Bjornstrand, Gu	Jacobsson, Ulla	Bergman, Ing	58	No	Bergman.gif
1987	98	Under the Sun of	Drama	Depardieu, Geri	Bonnaire, Sandi	Pialat, Maurice	45	No	NicholasCage.gif
1985	105	Vagabond	Drama	Meril, Macha	Bonnaire, Sandi	Varda, Agnes	49	No	NicholasCage.gif
1988	115	Working Girl	Comedy	Ford, Harrison	Griffith, Melanie	Nichols, Mike	25	No	NicholasCage.gif
1984	106	A Year of the Qui	Drama	Wilson, Scott	Komorowska, Iz	Zanussi, Krzys	78	No	NicholasCage.gif
1983	134	Yentl	Music	Patinkin, Mandi	Streisand, Barb	Streisand, Barb	46	No	NicholasCage.gif
1982	111	Yol	Drama	Akan, Tarik		Guney, Yilmaz	53	No	NicholasCage.gif
1992	102	The Addams Farr	Comedy	Julia, Raul	Huston, Anjelica	Sonnenfeld, B.	8	No	NicholasCage.gif
1992	88	Adventures in Din	Action	Katz, Omri	Hoffman, Shaw	Thompson, Bre	19	No	NicholasCage.gif
1992	95	Alan & Naomi	Drama	Haas, Lukas	Aquino, Vaness	Vanwagenen, S	3	No	NicholasCage.gif

# *Multi-Dimensional data*

- Each variable defines a data dimension
- Small # of dimensions easy
  - Data mapping, Cleveland's rules
- Easy stuff
  - Univariate: 1 variable
  - Bivariate : 2 variables
  - Trivariate: 3 variables

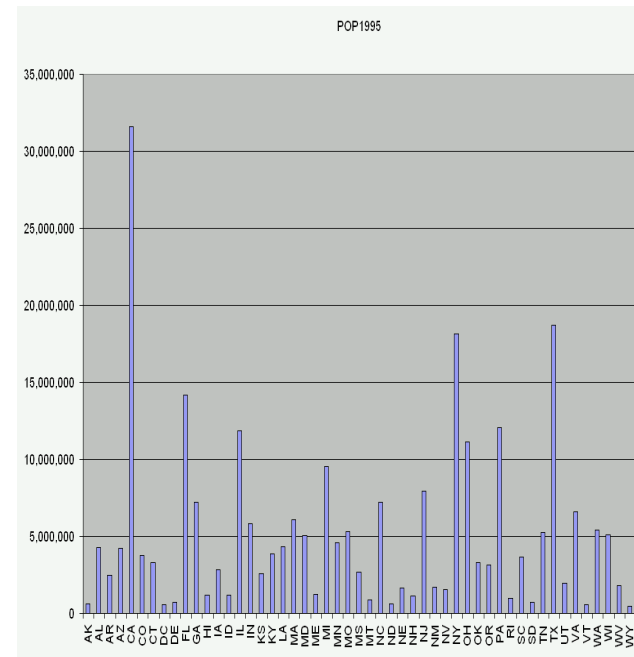
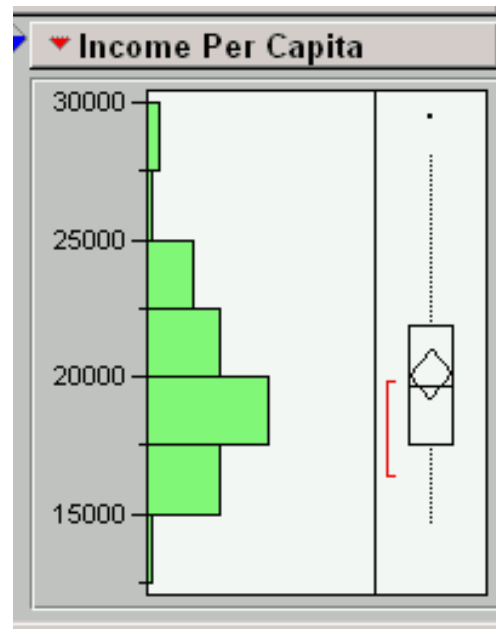
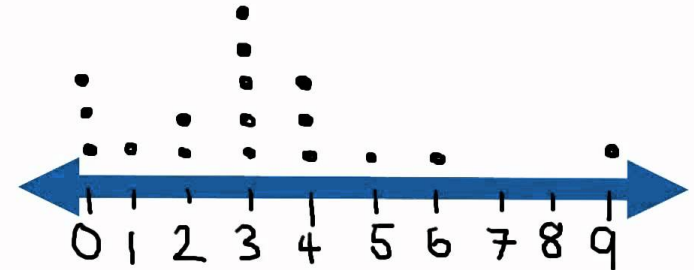


# Univariate

- Dot plot
- Bar chart
- Histogram

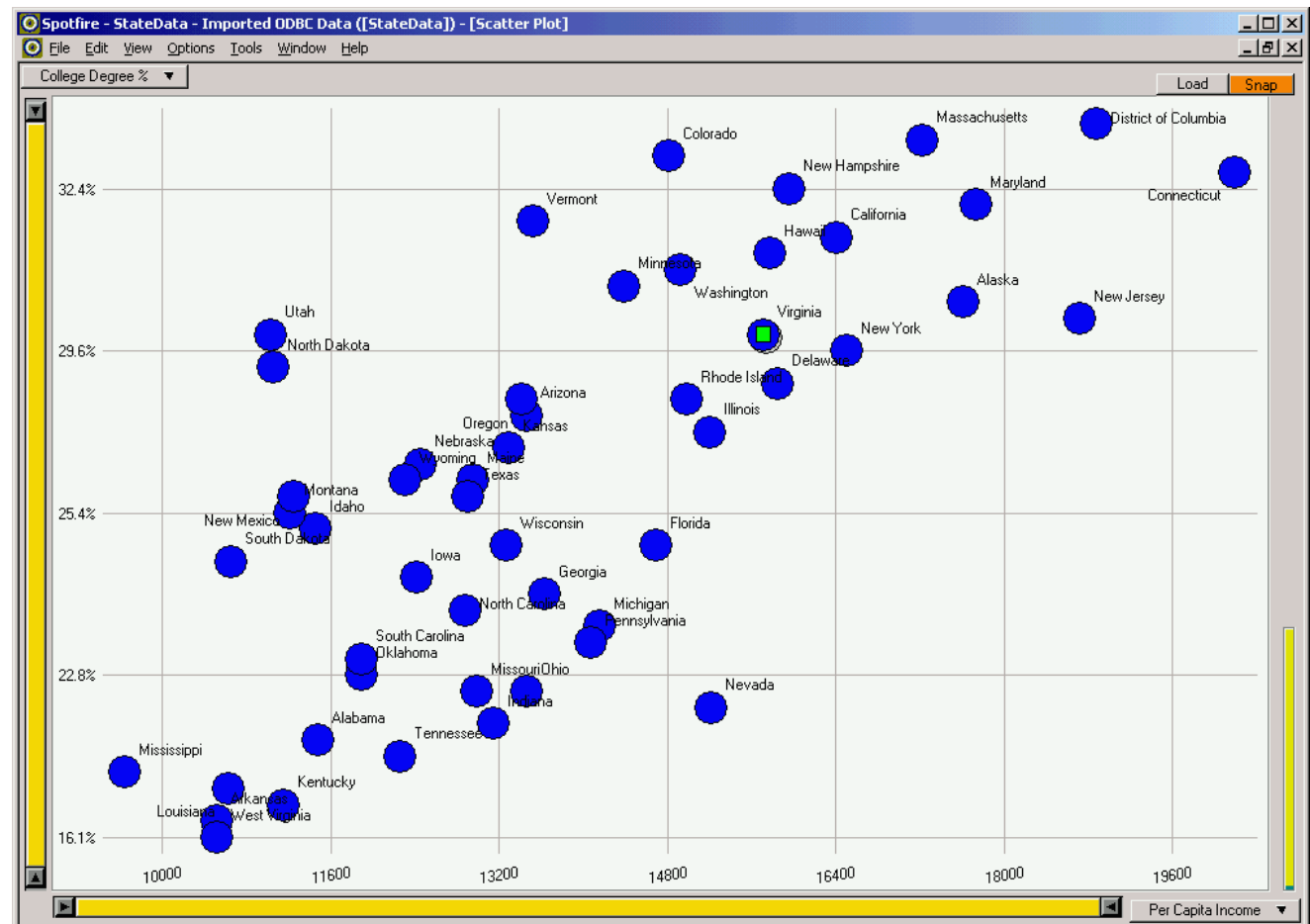
17 students we asked how many text messages they had sent on a particular day.

0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1



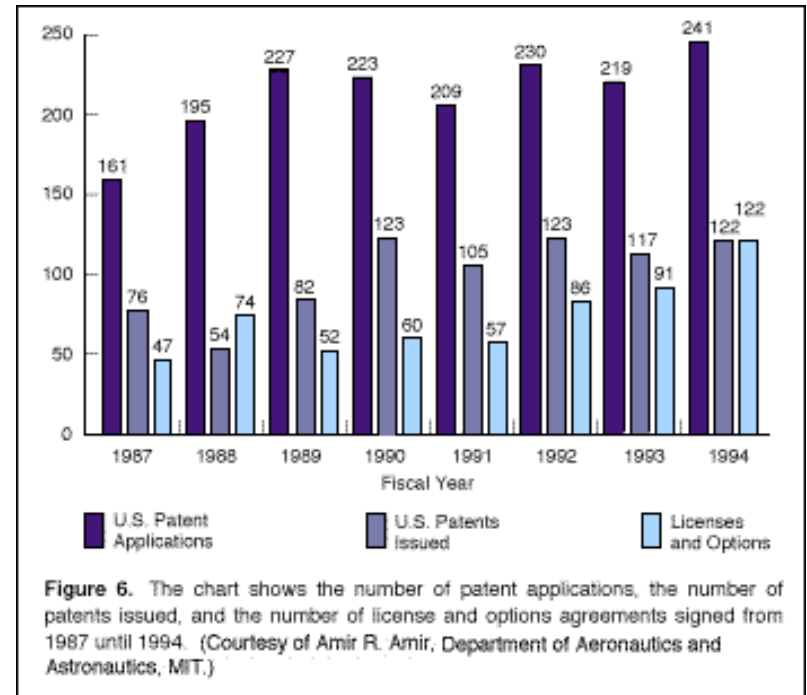
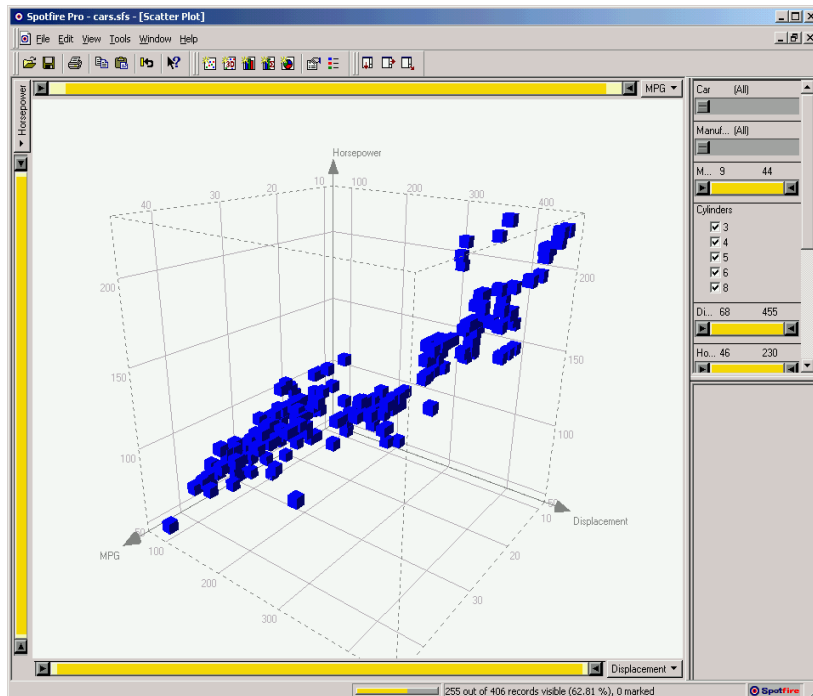
# Bivariate

- Scatterplot



# Trivariate

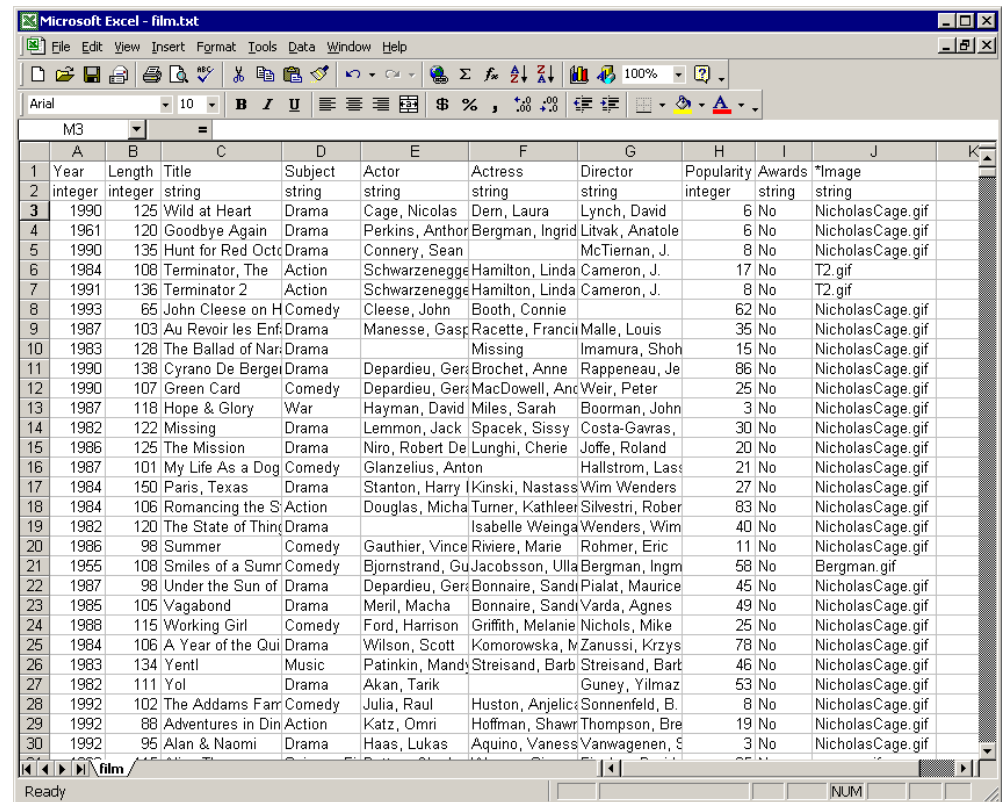
- 3D scatterplot, 3x barchart, 2D plot+color, 2D plot+size



# Multi-Dimensional data

- What about **many** dimensional data? n-D

What does 10-D space look like?

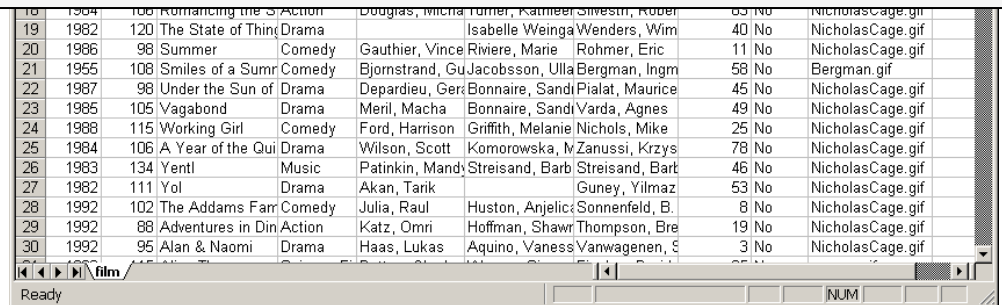


	A	B	C	D	E	F	G	H	I	J
1	Year	Length	Title	Subject	Actor	Actress	Director	Popularity	Awards	*Image
2	integer	integer	string	string	string	string	string	integer	string	string
3	1990	125	Wild at Heart	Drama	Cage, Nicolas	Dern, Laura	Lynch, David	6	No	NicholasCage.gif
4	1961	120	Goodbye Again	Drama	Perkins, Antho	Bergman, Ingrid	Litvak, Anatole	6	No	NicholasCage.gif
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10	1983	128	The Ballad of Nar	Drama		Missing	Imamura, Sho	15	No	NicholasCage.gif
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19	1982	120	The State of Thin	Drama		Isabelle Weinga	Wenders, Wim	40	No	NicholasCage.gif
20	1986	98	Summer	Comedy	Gauthier, Vince	Riviere, Marie	Rohmer, Eric	11	No	NicholasCage.gif
21	1955	108	Smiles of a Sumr	Comedy	Bjornstrand, Gu	Jacobsson, Ulla	Bergman, Ingm	58	No	Bergman.gif
22	1987	98	Under the Sun of	Drama	Depardieu, Ger	Bonnaire, Sandi	Pialat, Maunce	45	No	NicholasCage.gif
23	1985	105	Vagabond	Drama	Meril, Macha	Bonnaire, Sandi	Varda, Agnes	49	No	NicholasCage.gif
24	1988	115	Working Girl	Comedy	Ford, Harrison	Griffith, Melanie	Nichols, Mike	25	No	NicholasCage.gif
25	1984	106	A Year of the Qui	Drama	Wilson, Scott	Komorowska, M	Zanussi, Krzys	78	No	NicholasCage.gif
26	1983	134	Yentl	Music	Patinkin, Mand	Streisand, Barb	Streisand, Barb	46	No	NicholasCage.gif
27	1982	111	Yol	Drama	Akan, Tarik		Guney, Yilmaz	53	No	NicholasCage.gif
28	1992	102	The Addams Fam	Comedy	Julia, Raul	Huston, Anjelic	Sonnenfeld, B.	8	No	NicholasCage.gif
29	1992	88	Adventures in Din	Action	Katz, Omri	Hoffman, Shawr	Thompson, Bre	19	No	NicholasCage.gif
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# Multi-Dimensional data

- What about **many** dimensional data?  $n$ -D

*Note the difference between spatial dimensions (1,2,3D) and data dimensions (n-D)*



18	1984	106	Romancing the Stone	Action	Douglas, Micha	Turner, Kameel	Sivestri, Rober	65	No	NicholasCage.gif
19	1982	120	The State of Thin	Drama		Isabelle Weinga	Wenders, Wim	40	No	NicholasCage.gif
20	1986	98	Summer	Comedy	Gauthier, Vince	Riviere, Marie	Rohmer, Eric	11	No	NicholasCage.gif
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# *Terminology ...*

- Studied long before computer science, by statisticians and psychologists.
- *The term «dimensionality» is overloaded:*  
multidimensional, multivariate, hyperdimensional, hypervariate ...

# *Terminology ...*

- Variables can be dependent or independent:

$$y = f(x)$$

*y: **dependent** variable*

*x: **independent** variable*

# *Terminology ...*

- Variables can be dependent or independent:

$$y = f(x)$$

*y: **dependent** variable*

*x: **independent** variable*

- **Multidimensionality** refers to the dimensionality of the **independent** variables
- **Multivariate** refers to the dimensionality of the **dependent** variables



# *Multivariate Multi-Dimensional Example*

- Three dimensional cube showing temperature and pressure at various locations is **3d2v**
  - **3 independent variables:** XYZ
  - **2 dependent variables:** temperature and pressure

# *Warning!*

- We do not always know the dependent / independent variables
- For the sake of simplicity, we use the term “multi-dimensional” to refer to dimensionality of both dependent and independent variables
  - multivariate = multi-dimensional

**3d2v → 5D**

# *Genome Databases*



Challenging to Visualize

# Customer Transactions Data



Product ID	# of instances of cart add	# of unique instances of cart add	cart add Total Rs. Value	# of instances of cart removal	# of Unique instances of cart removal	cart remove Total value in Rs.	product views	unique product views	Value of products view in Rs.	Item revenue in Rs.	cart add % to product view	cart remove % to cart add
134454	455	381	461745	27	25	28773	14167	11540	14339133	52985.43	6%	33
134455	833	622	425667	8	5	3992	11171	8718	5714329	23819.47	1%	38
134456	435	340	570065	9	6	11691	12198	10215	16032202	80380.28	2%	47
134457	122	101	121878	2	2	1998	6926	6017	6919074	4415.45	2%	18
134458	239	196	609761	2	2	4998	11250	9733	28343750	57435.56	1%	54
134459	293	235	206307	8	7	5592	7171	6140	5044929	10508.89	3%	29
134460	314	249	288586	24	20	21576	6806	5275	6247294	32582.69	8%	42
134461	88	78	43912	0	0	0	4912	4324	2451088	882.2	0%	9
134462	256	210	342504	3	1	3987	3972	2901	5336358	27026.61	1%	86
134463	45	37	40455	0	0	0	5712	4424	5214288	10331.1	0%	7
134464	196	154	159604	36	29	29364	3638	2830	2948162	10594.49	18%	44
134465	90	67	227410	7	1	17493	5723	5005	14644277	11045.3	8%	40
134466	180	141	169470	6	4	5454	4939	4068	4645161	15272.2	3%	34
134467	26	19	10894	0	0	0	3134	2845	1321426	370.65	0%	3
134468	173	128	279737	4	3	6436	3023	2142	4888847	19916.26	2%	93
134469	166	136	267784	4	2	6436	2706	1937	4386384	12803.31	2%	99
134470	196	149	264474	3	2	3987	2840	2141	3810840	16450.98	2%	93
134471	36	27	26364	4	2	2796	3200	2753	2402490	5747	11%	8
134472	44	36	21956	0	0	0	3391	3032	1692109	882.2	0%	6
134473	116	87	159500	3	3	4125	1821	1457	2503123	0	3%	88
134474	35	25	17465	0	0	0	3413	2552	1703087	2205.51	0%	5

# Personal Data – Fitness Bands



Date	Activity	Group	Start	End	Duration	Distance	Steps	Calories
5/10/14	walking	walking	2014-05-10T09:34:04-04:00	2014-05-10T09:39:40-04:00	336	0.257	552	0
5/10/14	walking	walking	2014-05-10T10:00:00-04:00	2014-05-10T10:00:41-04:00	41	0.02	63	0
5/10/14	walking	walking	2014-05-10T12:33:26-04:00	2014-05-10T12:35:59-04:00	153	0.111	238	0
5/10/14	walking	walking	2014-05-10T12:46:41-04:00	2014-05-10T12:50:19-04:00	218	0.148	317	0
5/10/14	walking	walking	2014-05-10T13:00:47-04:00	2014-05-10T13:10:03-04:00	556	0.371	796	0
5/10/14	walking	walking	2014-05-10T14:04:51-04:00	2014-05-10T14:07:22-04:00	151	0.071	152	0
5/10/14	walking	walking	2014-05-10T15:09:11-04:00	2014-05-10T15:15:57-04:00	406	0.279	599	0
5/10/14	walking	walking	2014-05-10T16:03:50-04:00	2014-05-10T16:11:30-04:00	460	0.319	683	0
5/10/14	walking	walking	2014-05-10T17:09:15-04:00	2014-05-10T17:10:46-04:00	91	0.034	109	0
5/10/14	walking	walking	2014-05-10T18:05:09-04:00	2014-05-10T18:09:44-04:00	275	0.204	437	0
5/10/14	walking	walking	2014-05-10T19:04:54-04:00	2014-05-10T19:40:34-04:00	2140	1.373	2945	0
5/10/14	walking	walking	2014-05-10T20:00:00-04:00	2014-05-10T20:20:13-04:00	1213	0.808	1734	0
5/10/14	walking	walking	2014-05-10T21:24:02-04:00	2014-05-10T21:25:49-04:00	107	0.037	118	0
5/10/14	walking	walking	2014-05-10T23:18:10-04:00	2014-05-10T23:18:36-04:00	26	0.014	43	0
					6173	4.046	8786	0

After selecting variables, please click one of the above options

DIMENSION FILTERS

- Hierarchy**  
[Collapse all](#) | [Expand all](#) | [Unselect all](#)
- ▶ Income
  - ▶ Lending
  - ▶ Region
- 
- ▶ *Aggregates*

- ▶ COUNTRY (Available: 214 | Selected: 1)
- ▶ SERIES (Available: 1300 | Selected: 0)
- ▶ TIME (Available: | Selected: 0)

YOUR CURRENT SELECTION

- ▼ DATABASE  
 World Development Indicators  
[Change database](#)

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- ▼ COUNTRY (1)  
[Remove all](#) | [Sort](#)  
*Drag to rearrange the order*
  - ✖ France

---

- ▶ SERIES (0)

---

- ▶ TIME (0)

After selecting variables, please click one of the above options

DIMENSION FILTERS

Hierarchy Collapse all | Expand all

- Topic
  - Education
  - Environment
  - Economic Policy & Debt
  - Financial Sector
  - Health
  - Infrastructure
  - Labor & Social Protection
  - Poverty
  - Private Sector & Trade
  - Public Sector

COUNTRY (Available:214| Selected:1)

SERIES (Available:1300| Selected:0)

Type keywords to filter

( | A | B | C | D | E | F | G | H | I | L | M | N | O | P | Q | R | S | T | U | V | W | )

Select all | Unselect all | Invert selection

- ( )
- (%) Benefits held by 1st 20% population - All Social Insurance
- (%) Benefits held by 1st 20% population - All Social Protection
- (%) Benefits held by 1st 20% population - All Social Safety Nets
- (%) Benefits held by 1st 20% population - Unemp benefits and ALMP
- (%) Generosity of All Social Insurance
- (%) Generosity of All Social Protection
- (%) Generosity of All Social Safety Nets
- (%) Generosity of Unemp benefits and ALMP
- (%) Program participation - All Social Insurance
- (%) Program participation - All Social Protection
- (%) Program participation - All Social Safety Nets
- (%) Program participation - Unemp benefits and ALMP
- A
- ARI treatment (% of children under 5 taken to a health provider)
- Access to electricity (% of population)
- Adjusted net enrollment rate, primary (% of primary school age children)
- Adjusted net enrollment rate, primary, female (% of primary school age children)
- Adjusted net enrollment rate, primary, male (% of primary school age children)
- Adjusted net national income (annual % growth)
- Adjusted net national income (constant 2005 US\$)
- Adjusted net national income (current US\$)
- Adjusted net savings, excluding particulate emission damage (% of GNI)
- Adjusted net savings, excluding particulate emission damage (current US\$)
- Adjusted net savings, including particulate emission damage (% of GNI)

TIME (Available:| Selected:0)

YOUR CURRENT SELECTION

DATABASE

World Development Indicators Change database

COUNTRY (1)

SERIES (0)

Remove all | Sort Drag to rearrange the order

TIME (0)





After selecting variables, please click one of the above options

DIMENSION FILTERS

Hierarchy

Collapse all | Expand all

Year

▶ COUNTRY (Available: 214 | Selected: 1)  
 ▶ SERIES (Available: 1300 | Selected: 0)  
 ▼ TIME (Available: 54 | Selected: 0)

SELECT YEARS FUNCTIONS

Availability Range: **Year 1960 - 2013** Please drag the slider handle below to select the range

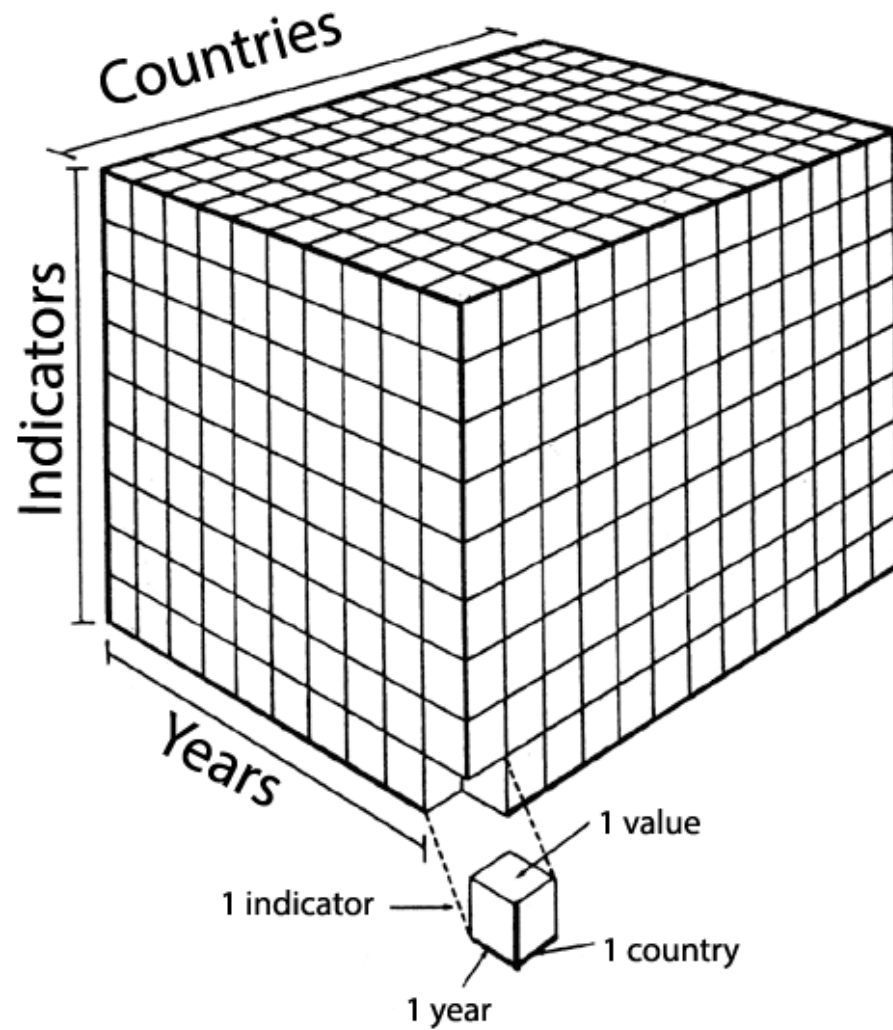
Select all | Unselect all | Invert selection

- 2013
- 2012
- 2011
- 2010
- 2009
- 2008
- 2007
- 2006
- 2005
- 2004
- 2003
- 2002
- 2001
- 2000
- 1999
- 1998
- 1997
- 1996
- 1995
- 1994
- 1993
- 1992
- 1991

YOUR CURRENT SELECTION

- ▼ DATABASE
  - World Development Indicators
  - [Change database](#)
- ▶ COUNTRY (1)
- ▶ SERIES (0)
- ▼ TIME (0)
  - [Remove all](#) | [Sort](#) |  Ascending

# *the data cube*



# *multi-dimensional data – our view today*

	Benefits	Generosity	Participation	Savings	Interest	Bird species	Birth Rate	C02 Emissions	...
Afghanistan									
Albania									
Algeria									
Andorra									
Angola									
Argentina									
Armenia									
Aruba									
Australia									
...									

The diagram illustrates a multi-dimensional data matrix. A large orange arrow starts at the top-left cell (Afghanistan, Benefits) and points to the right, then down, and then right again, forming an L-shape. This arrow highlights the dimensions of the data. A grey box labeled "d-dimensions" is positioned above the arrow's horizontal segment, indicating the number of features (columns). Another grey box labeled "n data points (records)" is positioned to the left of the arrow's vertical segment, indicating the number of observations (rows). A third grey box labeled "n x d matrix" is positioned in the lower right area of the matrix, summarizing the overall structure.

***VISUALIZING MULTI-DIMENSIONAL  
DATA***

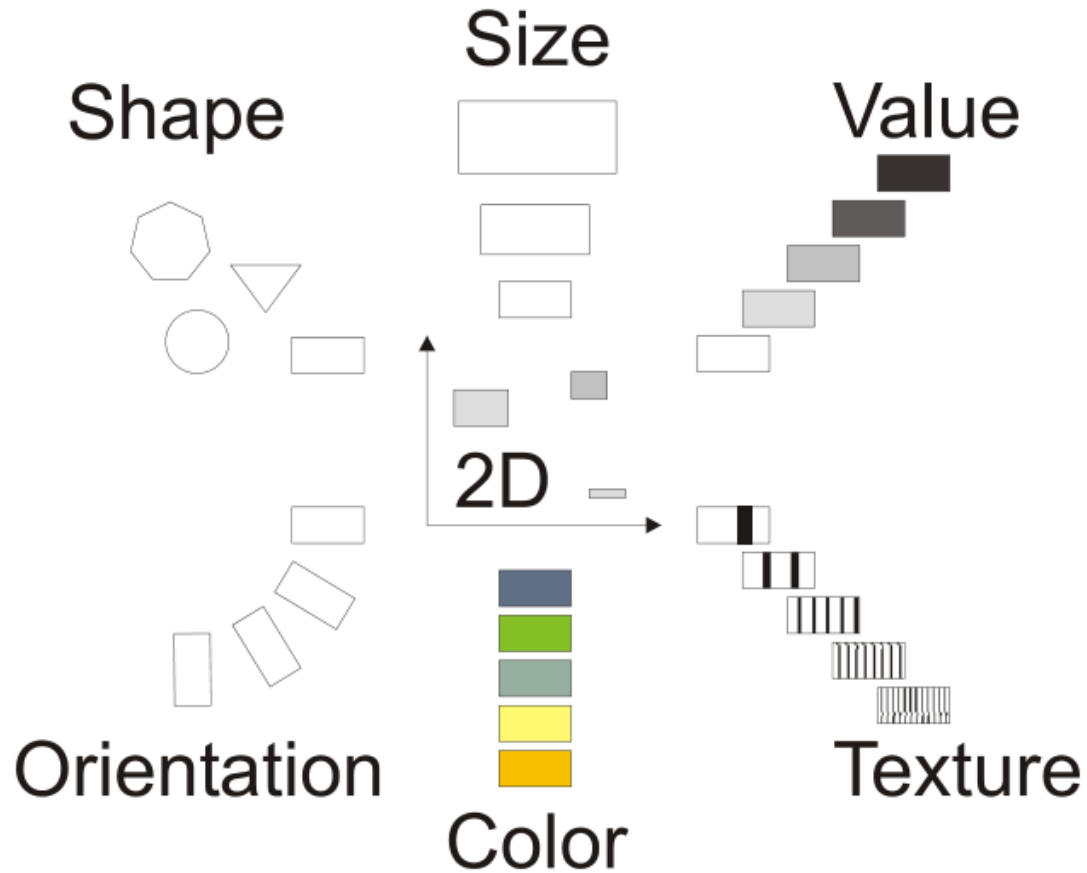
# *Data-Type Taxonomy*

- 1D: list, ...
- 2D: maps, ...
- 3D: volumes, ...
- Temporal: timelines , ...
- Hierarchies/trees: directories, ...
- Network/Graphs: web, ...
- **Multi-dimensional: databases, ...**

# *Challenges*

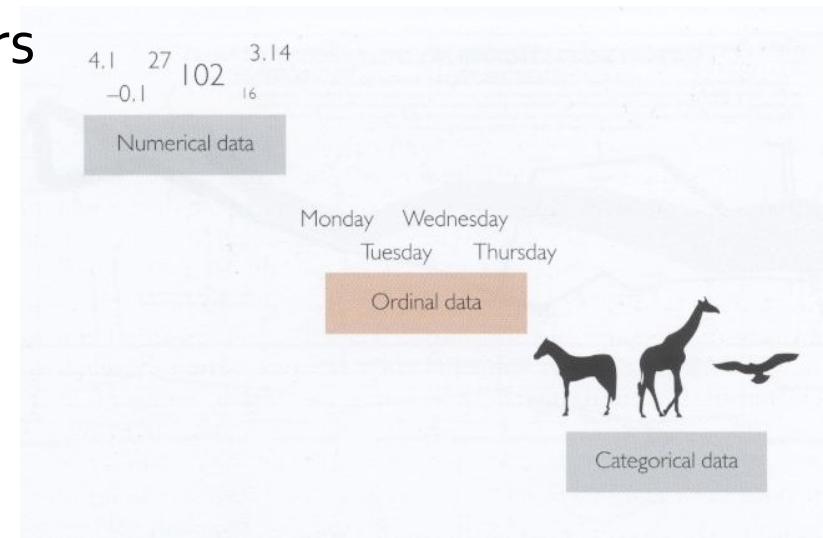
- Mapping from ND to 2D
- Clutter on screen
- Difficult user navigation in data space

# *recap – last lecture*



# recap: Variable Types

- **N-Nominal** (equal or not equal to other values)
  - Example: gender, hair color (blond, brown, black, red)
- **O-Ordinal** (obeys  $<$  relation, ordered set)
  - Example: soccer leagues, rainbow colors
- **Q-Quantitative** (can do math on them)
  - Example: age, Photoshop colors





# *recap - visual encodings*

## **Quantitative**

## **Ordinal**

## **Nominal**

Position

Length

Angle

Slope

Area

Volume

Density

Color Saturation

Color Hue

Texture

Connection

Containment

Shape



Position

Density

Color Saturation

Color Hue

Texture

Connection

Containment

Length

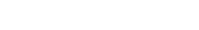
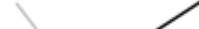
Angle

Slope

Area

Volume

Shape



Position

Color Hue

Texture

Connection

Containment

Density

Color Saturation

Shape

Length

Angle

Slope

Area

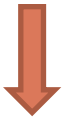
Volume

# *the problem*

1300 indicators



214 countries



	Benefits	Generosity	Participation	Savings	Interest	Bird species	Birth Rate	C02 Emissions	...
Afghanistan									
Albania									
Algeria									
Andorra									
Angola									
Argentina									
Armenia									
Aruba									
Australia									
...									

Hundreds or thousands of dimensions, only limited number of visual variables!

## *two main approaches*

- 1. Attempt to visualize all dimensions**
2. Visualize only parts of the data  
→ add interaction



*Interaction Lecture*

# *how do we visualize data points?*



Data  
Transformation

Spatial Mapping  
Transformation

Presentation  
Transformation

View  
Transformation



1. Find a Dataset

# *how do we visualize data points?*



Data  
Transformation

Spatial Mapping  
Transformation

Presentation  
Transformation

View  
Transformation



2. Transform the data

to arrive at data you want to visualize

# *transforming MD data*

- Filter the data
  - decide to show LESS of the data
  - in practice you almost always do this
- Use a dimensionality reduction technique
  - E.g. principal component analysis, multi-dimensional scaling
  - often used for exploring (dis)similarities in data

# *how do we visualize data points?*



Data  
Transformation

Spatial Mapping  
Transformation

Presentation  
Transformation

View  
Transformation



3. Your remaining  
possibly transformed dimensions

# *how do we visualize data points?*



Data  
Transformation

Spatial Mapping  
Transformation

Presentation  
Transformation

View  
Transformation



4. Choose how to put the data points on the drawing plane



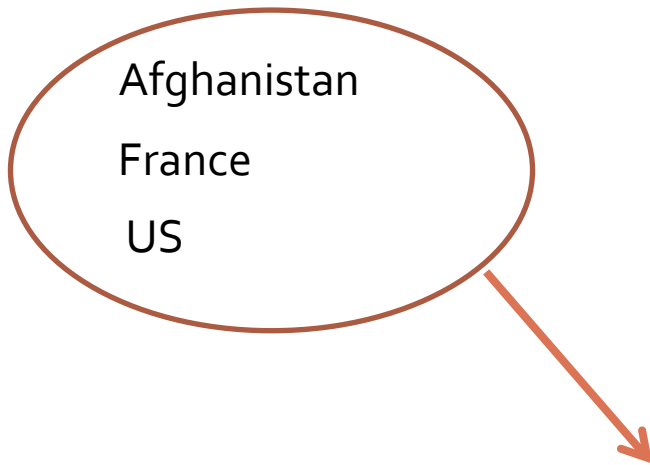
# 1. *find a layout*

- Let's start with one dimension: income per person
- 1-D Data points: Afghanistan, France, US

	Income per person
Afghanistan	850
France	29500
US	41000

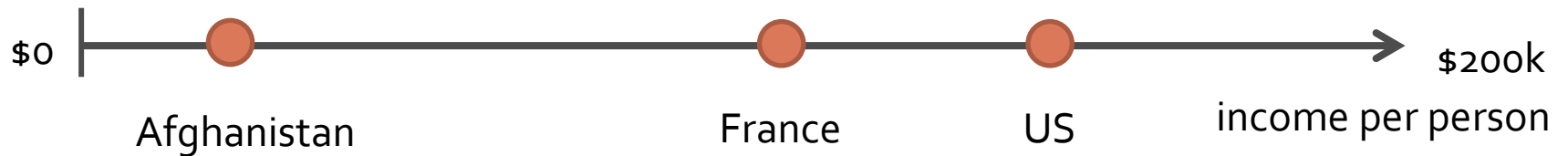
# 1. *find a layout*

	Income per person
Afghanistan	850
France	29500
US	41000



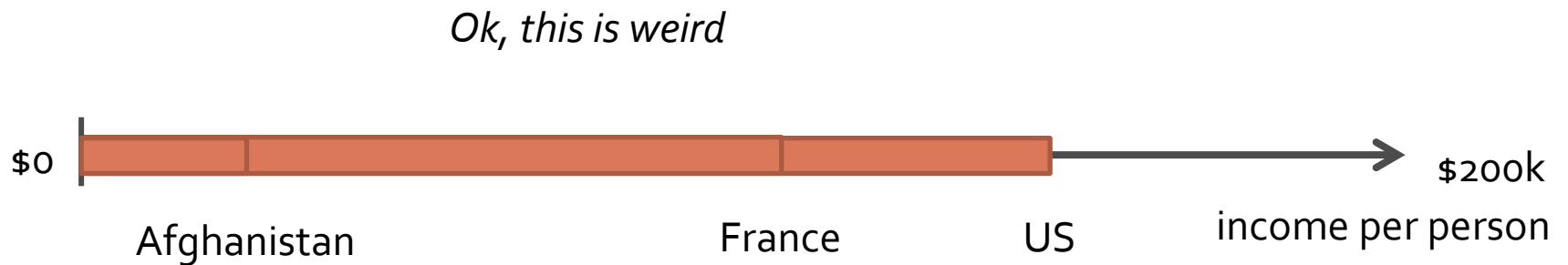
## *2. choose a visual encoding & mark*

- E.g. position + circle



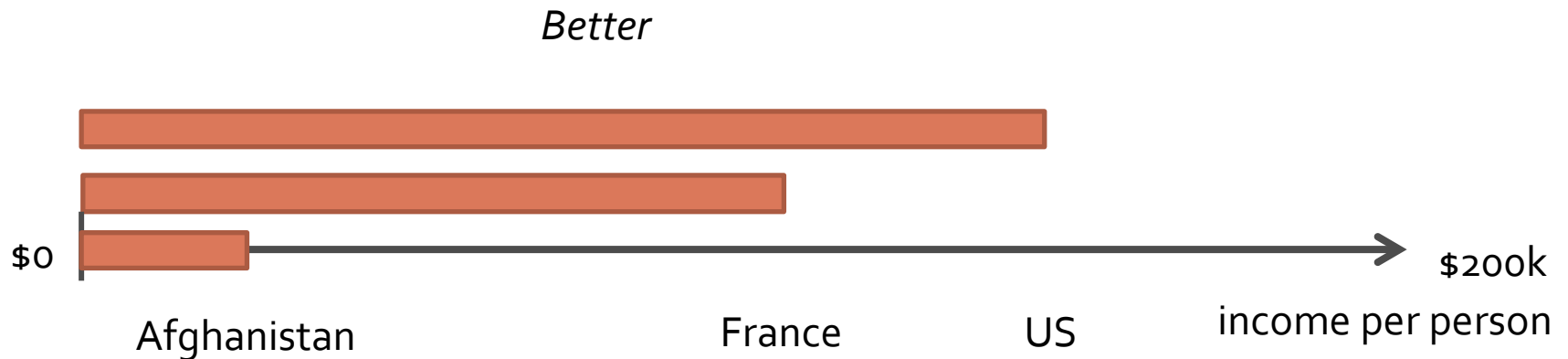
## 2. choose a visual encoding & mark

- E.g. length + rectangle



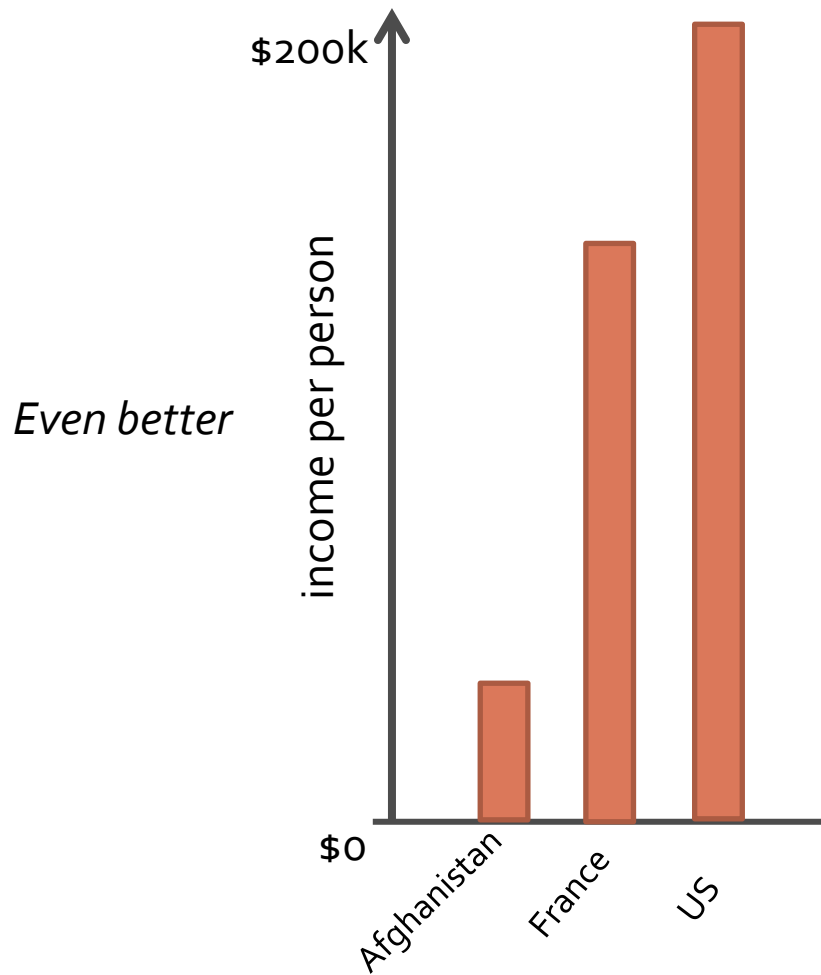
## 2. choose a visual encoding & mark

- E.g. length + rectangle



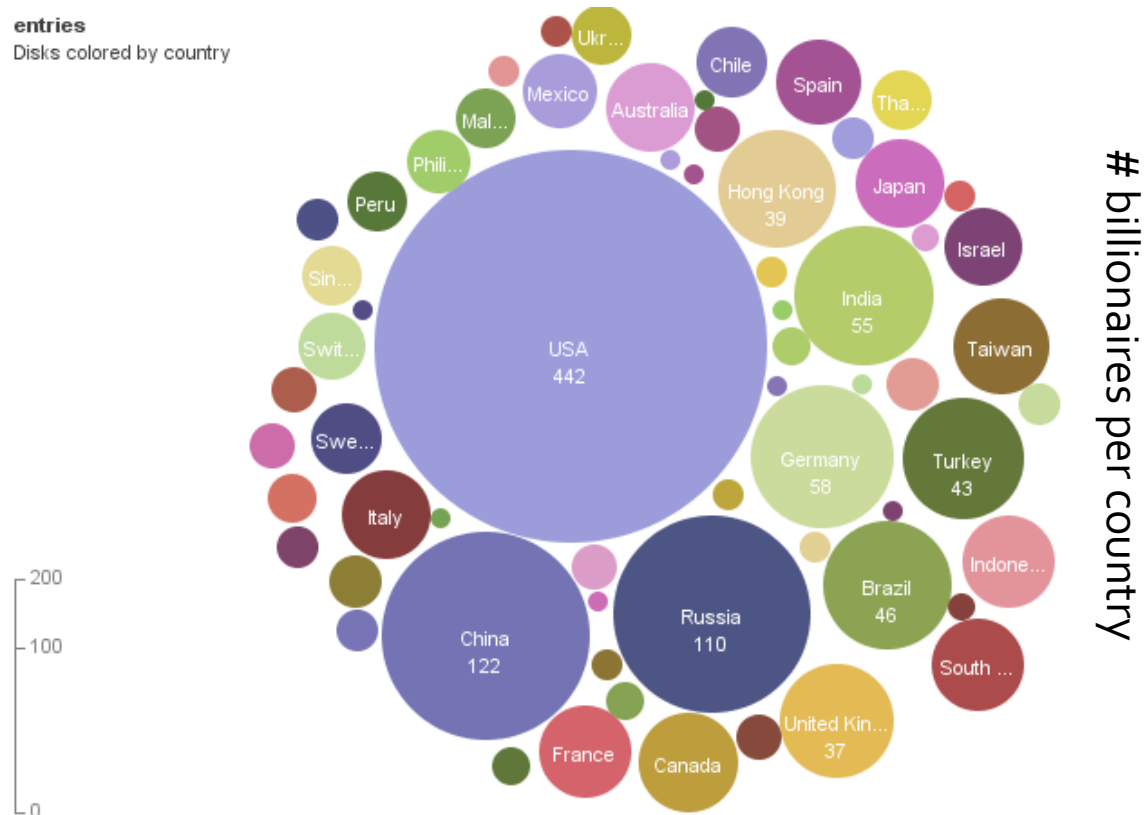
## 2. choose a visual encoding & mark

- E.g. length + rectangle

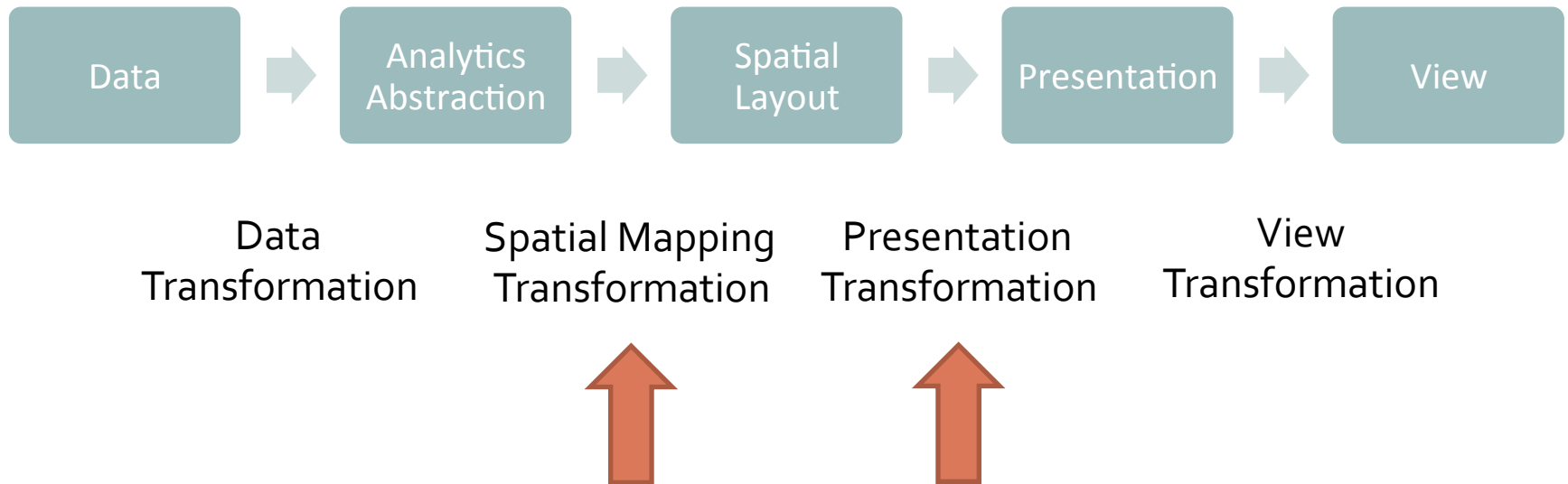


# *bubble chart*

- Spatial layout does not have to be ordered and can be computed by a variety of algorithms (what's most effective is another question)



*Choosing spatial layouts, marks, and visual encodings often goes hand-in-hand*



4. Choose how to put the data points on the drawing plane

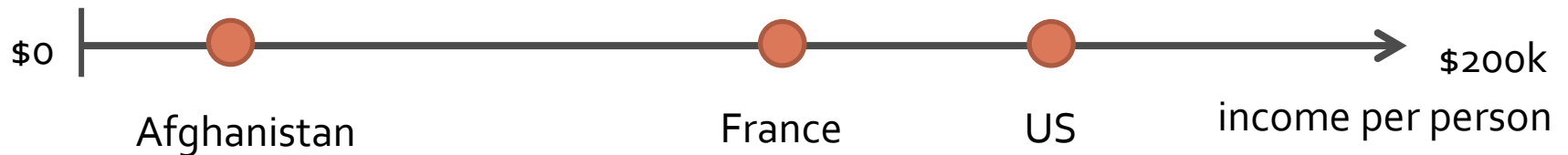
5. Choose additional visual encodings



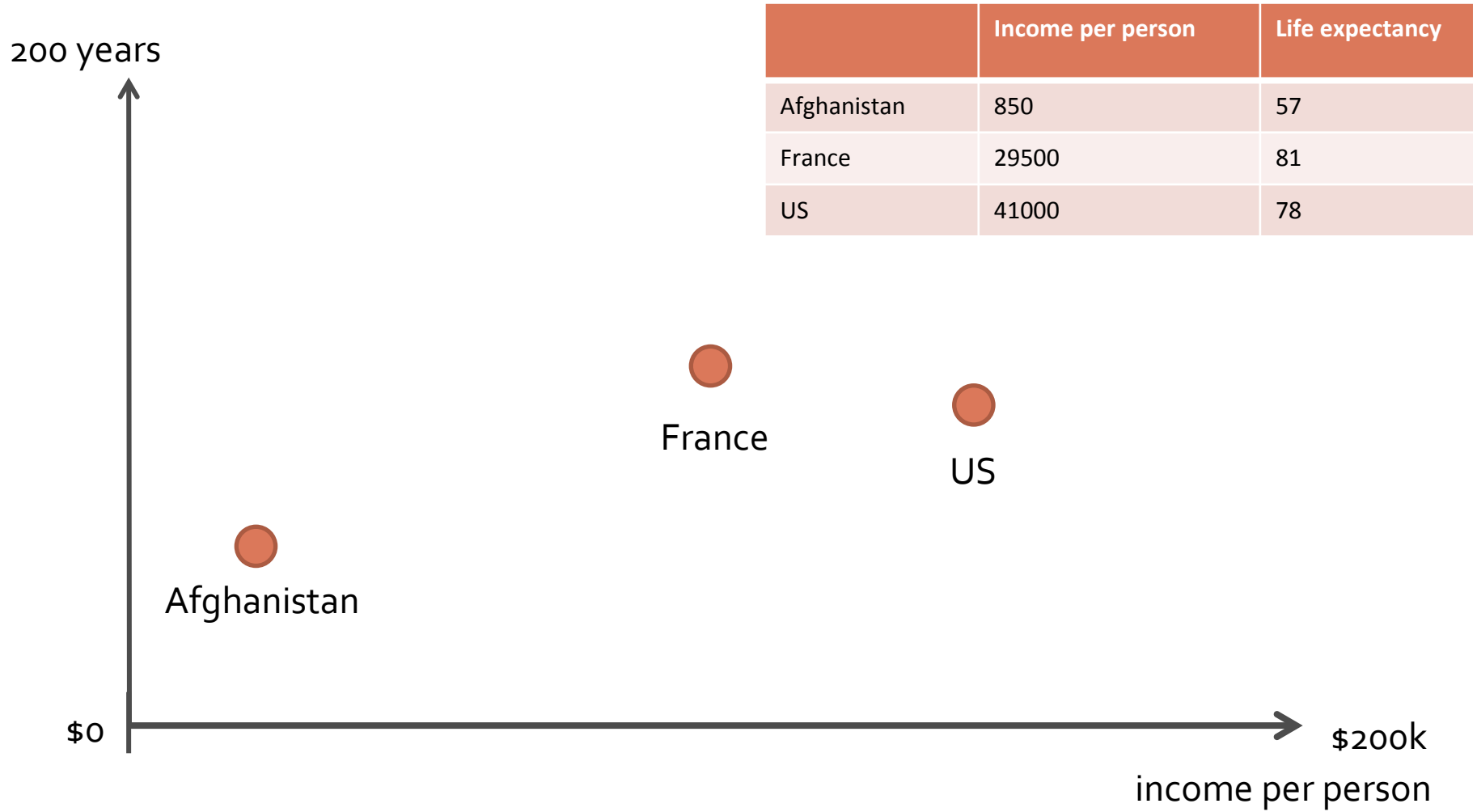
# 1. find a layout

	Income per person	Life expectancy
Afghanistan	850	57
France	29500	81
US	41000	78

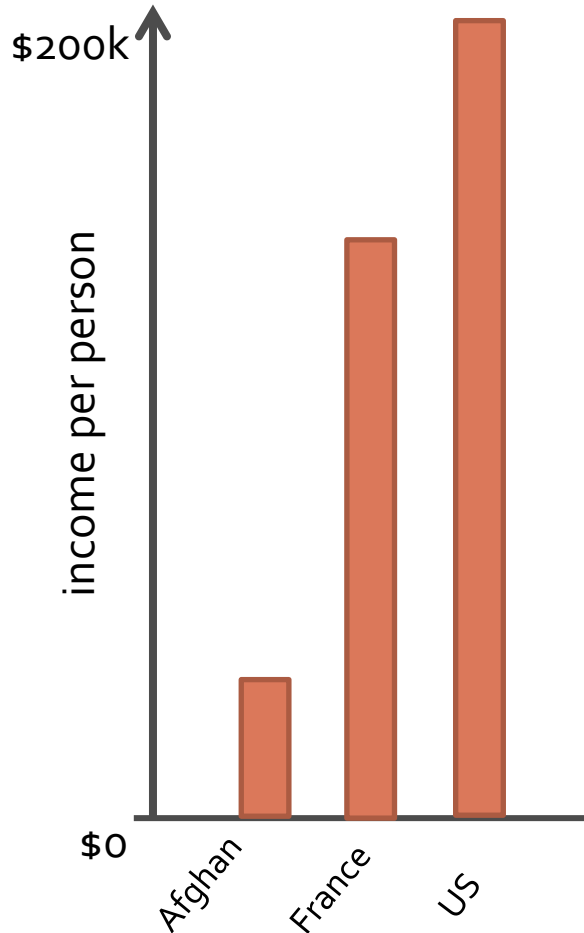
How do we extend this to 2 data dimensions?



# 1. find a layout



*how do you extend this to 2D?*



	Income per person	Life expectancy
Afghanistan	850	57
France	29500	81
US	41000	78

*Think about it!*

## *adding a third dimension*

	Income per person	Life expectancy	Children per Woman
Afghanistan	850	57	7.1
France	29500	81	1.9
US	41000	78	2.1

# adding a third dimension

200 years

*Suggestions?*

	Income per person	Life expectancy	Children per Woman
Afghanistan	850	57	7.1
France	29500	81	1.9
US	41000	78	2.1

France

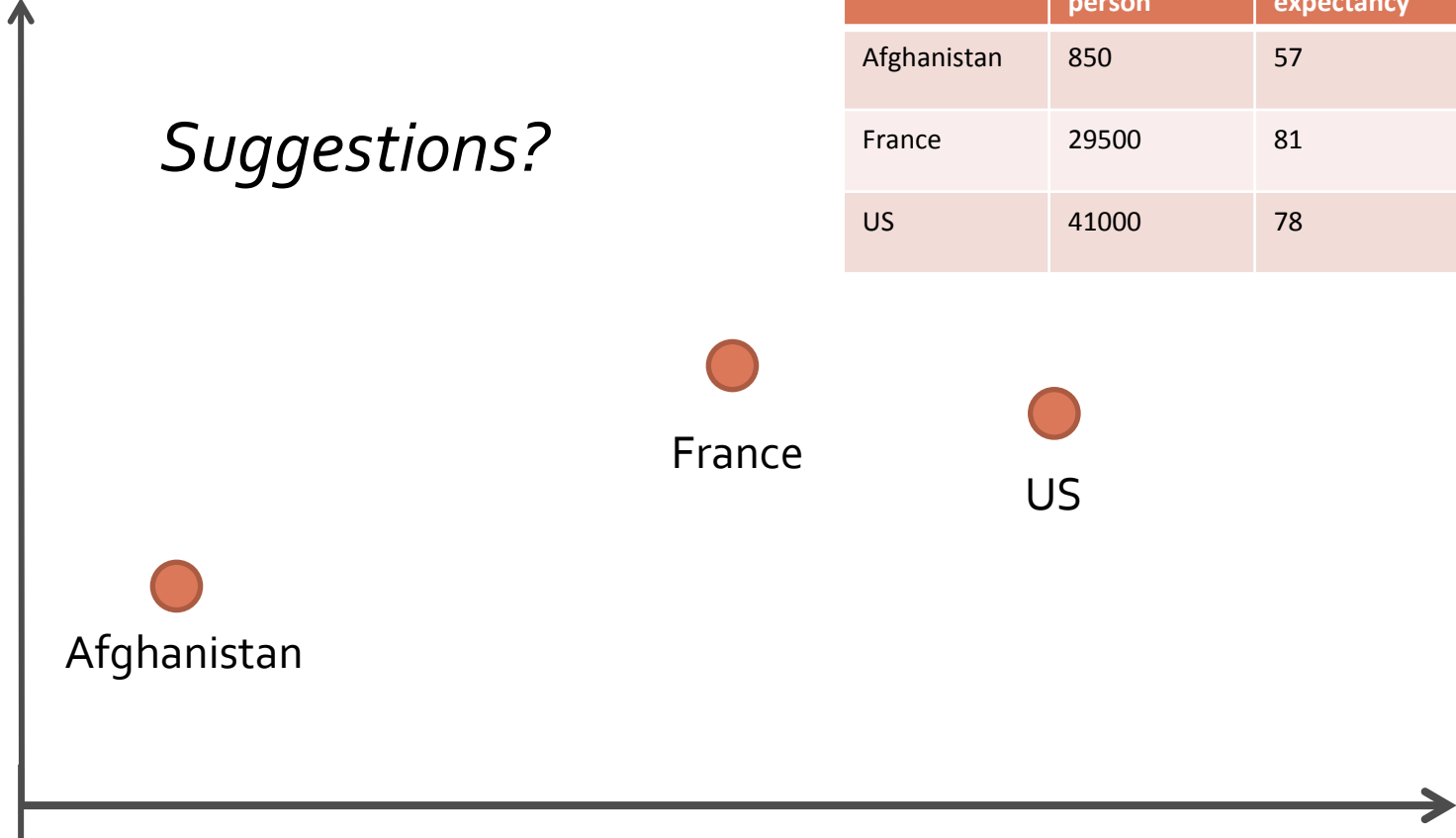
US

Afghanistan

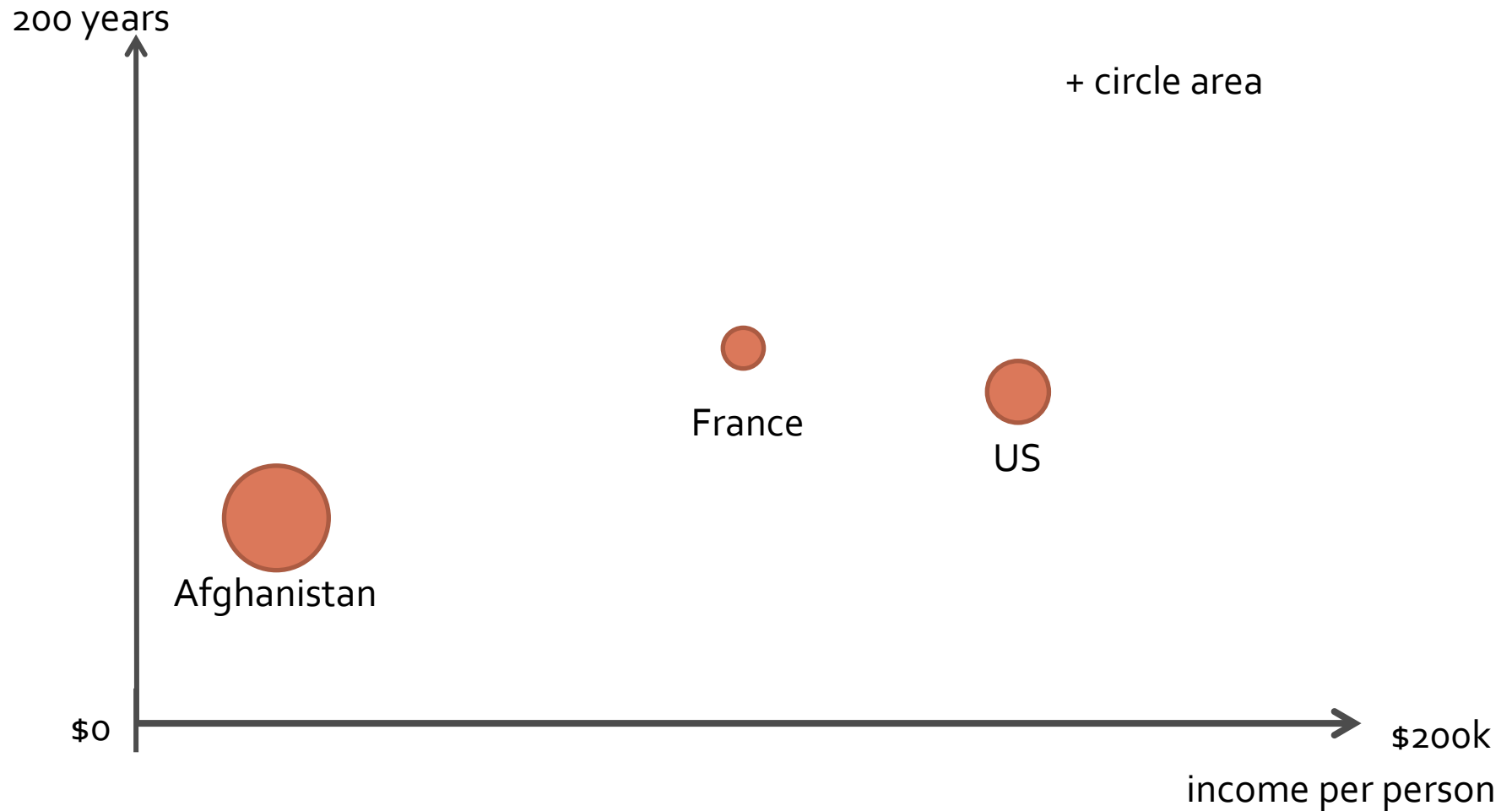
\$0

\$200k

income per person

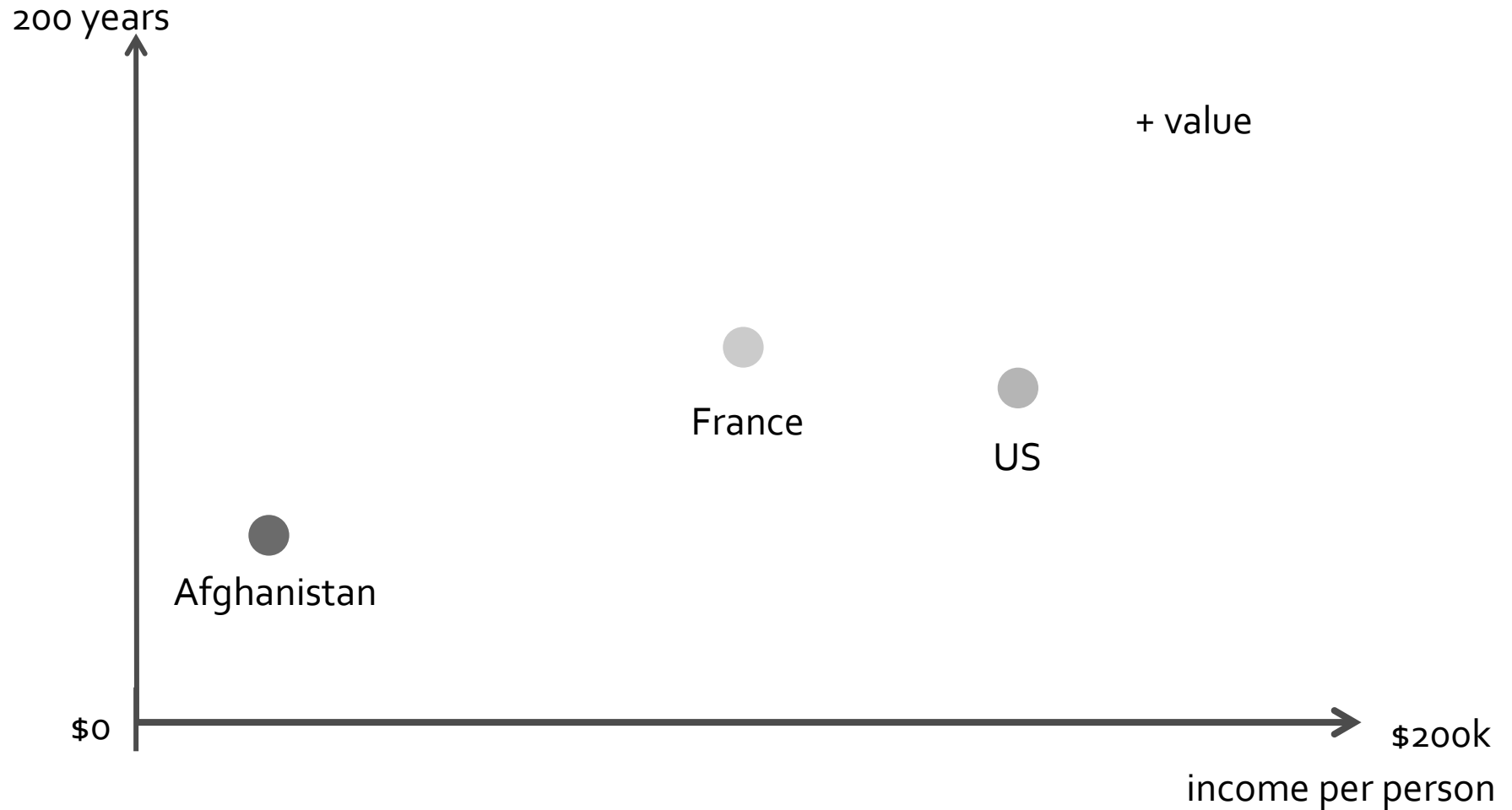


# *add another visual encoding*



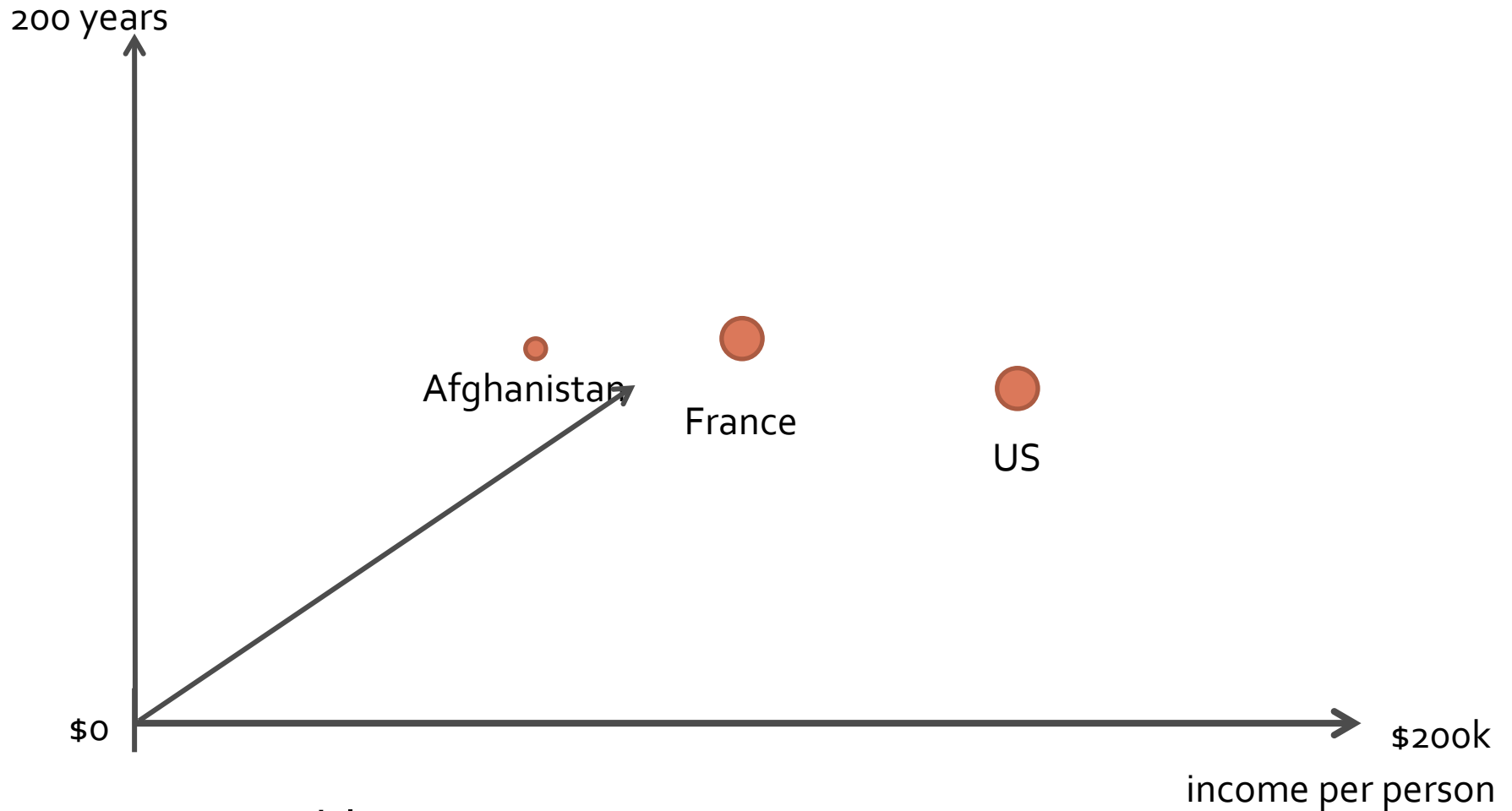
Problem:  
Does not scale well to more dimensions

# *add another visual encoding*



Problem:  
Does not scale well to more dimensions

*add an axis*



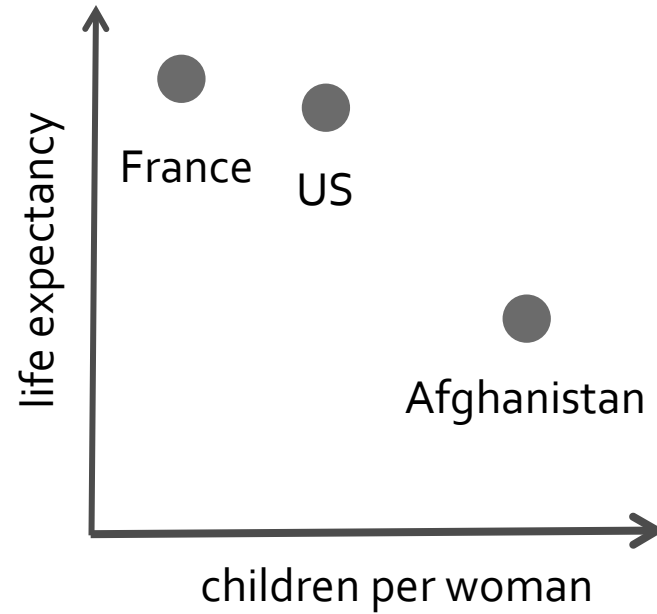
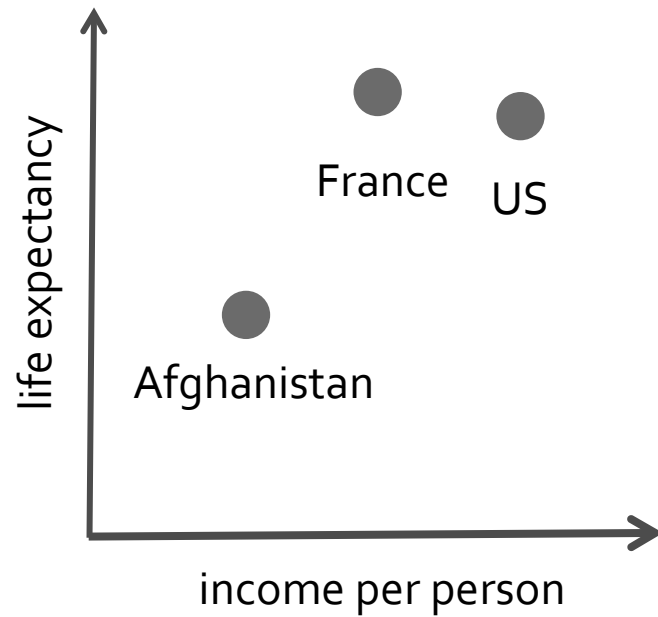
Problems:

Occlusion, perspective distortion, does not scale

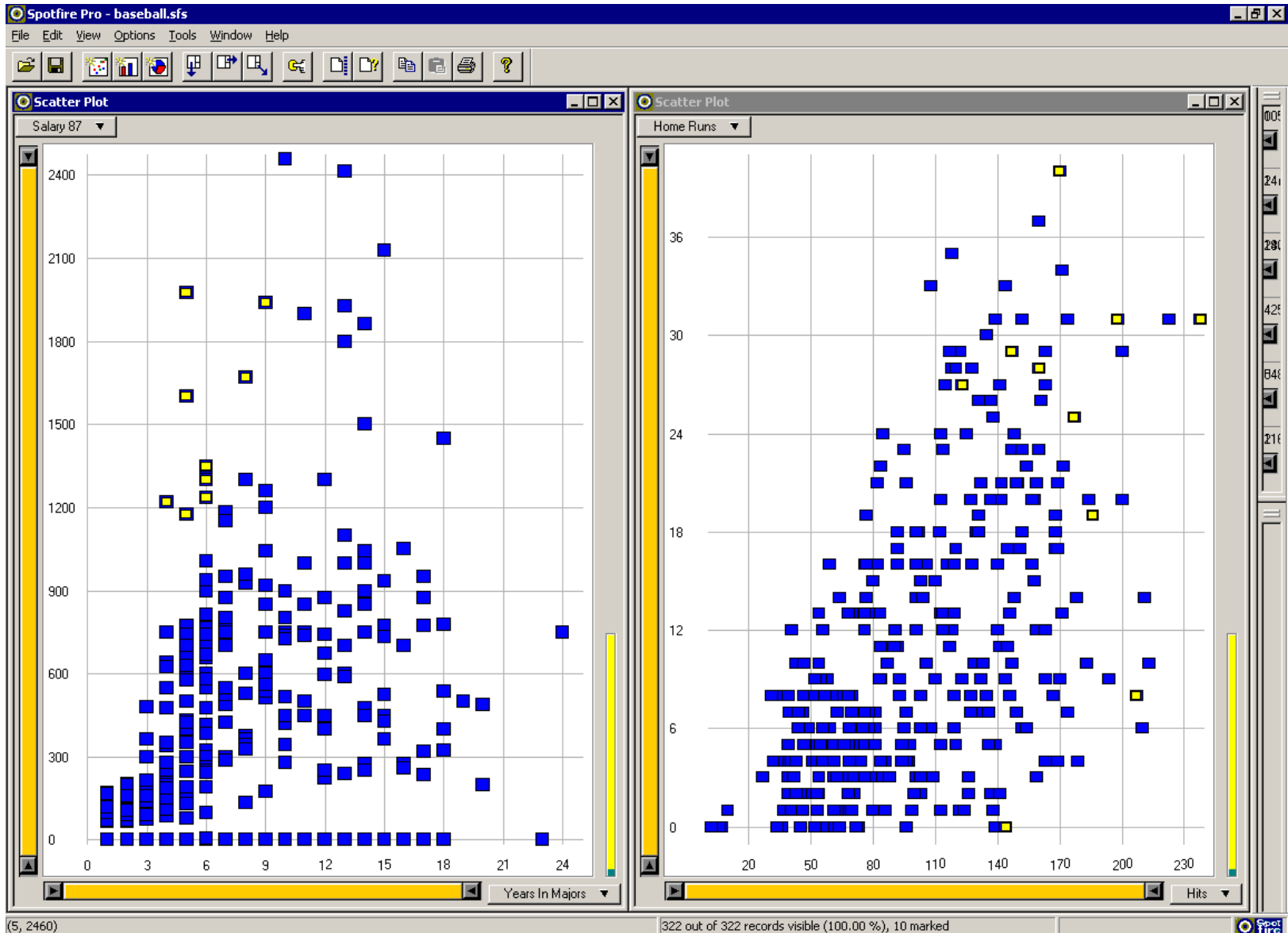
→ Not usually recommended



*add an axis*



# Linked Views with Brushing and Linking



*back to our original problem: 1. find a layout*

- We can also decide to display the whole data table and augment it

# displaying the data table itself

- Heatmap on individual cells
- (Excel does this)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	Rank	Player	Team	Pos	G	AB	R	H	2B	3B	HR	RBI	BB	SO	SB	CS	AVG	OBP	SLG	OPS
2	1	Posey, B	SF	C	148	530	78	178	39	1	24	103	69	96	1	1	0.336	0.408	0.549	0.957
3	2	Cabrera, M	DET	3B	161	622	109	205	40	0	44	139	66	98	4	1	0.33	0.393	0.606	0.999
4	3	McCutchen, A	PIT	CF	157	593	107	194	29	6	31	96	70	132	20	12	0.327	0.4	0.553	0.953
5	4	Trout, M	LAA	CF	139	559	129	182	27	8	30	83	67	139	49	5	0.326	0.399	0.564	0.963
6	5	Beltre, A	TEX	3B	156	604	95	194	33	2	36	102	36	82	1	0	0.321	0.359	0.561	0.921
7	6	Braun, R	MIL	LF	154	598	108	191	36	3	41	112	63	128	30	7	0.319	0.391	0.595	0.987
8	7	Mauer, J	MIN	C	147	545	81	174	31	4	10	85	90	88	8	4	0.319	0.416	0.446	0.861
9	8	Jeter, D	NYN	SS	159	683	99	216	32	0	15	58	45	90	9	4	0.316	0.362	0.429	0.791
10	9	Molina, Y	STL	C	138	505	65	159	28	0	22	76	45	55	12	3	0.315	0.373	0.501	0.874
11	10	Felder, P	DET	1B	162	581	83	182	33	1	30	108	85	84	1	0	0.313	0.412	0.528	0.94
12	11	Hunter, T	LAA	CF	140	534	81	167	24	1	16	92	38	133	9	1	0.313	0.365	0.451	0.817
13	12	Butler, B	KC	1B	161	614	72	192	32	1	29	107	54	111	2	1	0.313	0.373	0.51	0.882
14	13	Cano, R	NYN	2B	161	627	105	196	48	1	33	94	61	96	3	2	0.313	0.379	0.55	0.929
15	14	Pacheco, J	COL	3B	132	475	51	147	32	3	5	54	22	61	7	2	0.309	0.341	0.421	0.762
16	15	Craig, A	STL	RF	119	469	76	144	35	0	22	92	37	89	2	1	0.307	0.354	0.522	0.876
17	16	Scutaro, M	SF	3B	156	620	87	190	32	4	7	74	40	49	9	4	0.306	0.348	0.405	0.753
18	17	Wright, D	NYM	3B	156	581	91	178	41	2	21	93	81	112	15	10	0.306	0.391	0.492	0.883
19	18	Jay, J	STL	CF	117	443	70	135	22	4	4	40	34	71	19	7	0.305	0.373	0.4	0.773
20	19	Murphy, D	TEX	LF	147	457	65	139	29	3	15	61	54	74	10	5	0.304	0.38	0.479	0.859
21	20	Rios, A	CWS	CF	157	605	93	184	37	8	25	91	26	92	23	6	0.304	0.334	0.516	0.85
22	21	Gonzalez, C	COL	LF	135	518	89	157	31	5	22	85	56	115	20	5	0.303	0.371	0.51	0.881

Imagesource: dataremixed.com

# table lens

Ramana Rao, Stuart K. Card, Peter Pirolli (Xerox Parc) – 1994-96

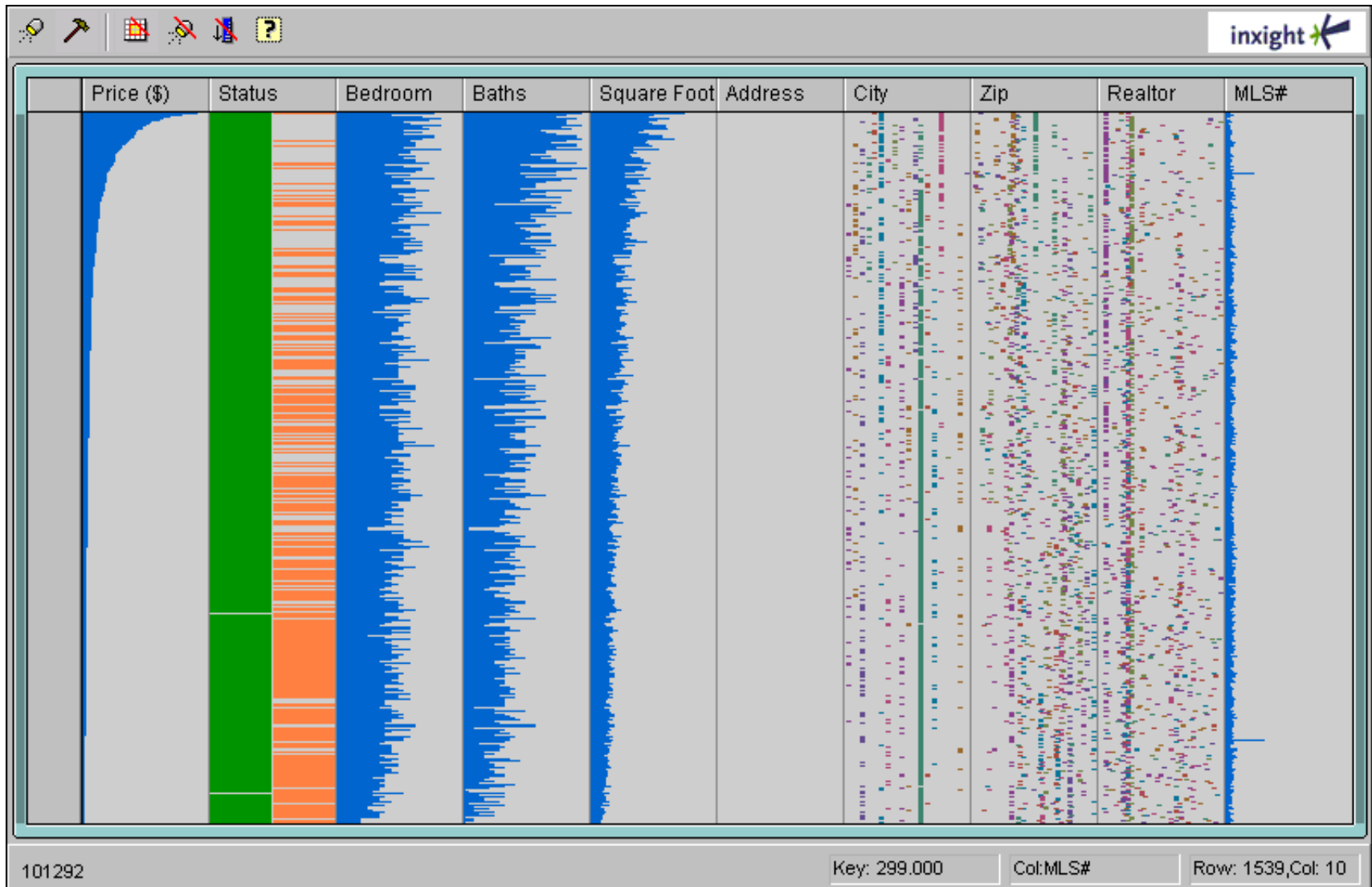


Image source: <http://www.sapdesignguild.org/goodies/controls/TableLens.htm>

# *Classification of MD Vis Techniques*

- Geometric
- Icon-based
- Pixel-oriented
- Hierarchical
- Graph-based

# *Classification of MD Vis Techniques*

- Geometric
- Icon-based
- Pixel-oriented
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- Graph-based

# Classification of MD Vis Techniques

## Geometric techniques



# *scatterplot matrix*

- This idea scales relatively well

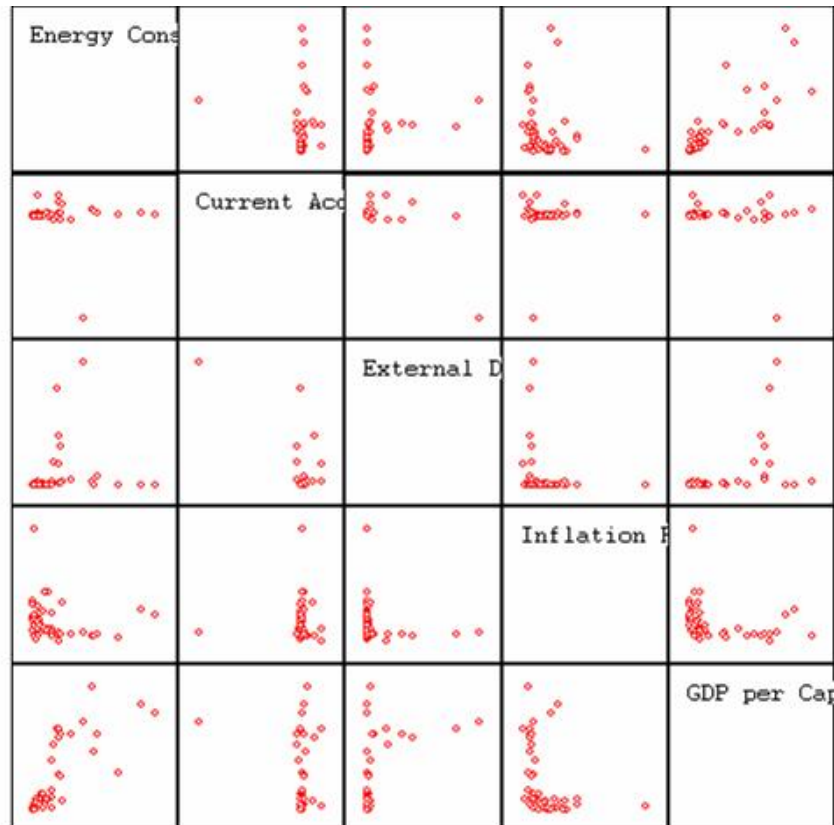
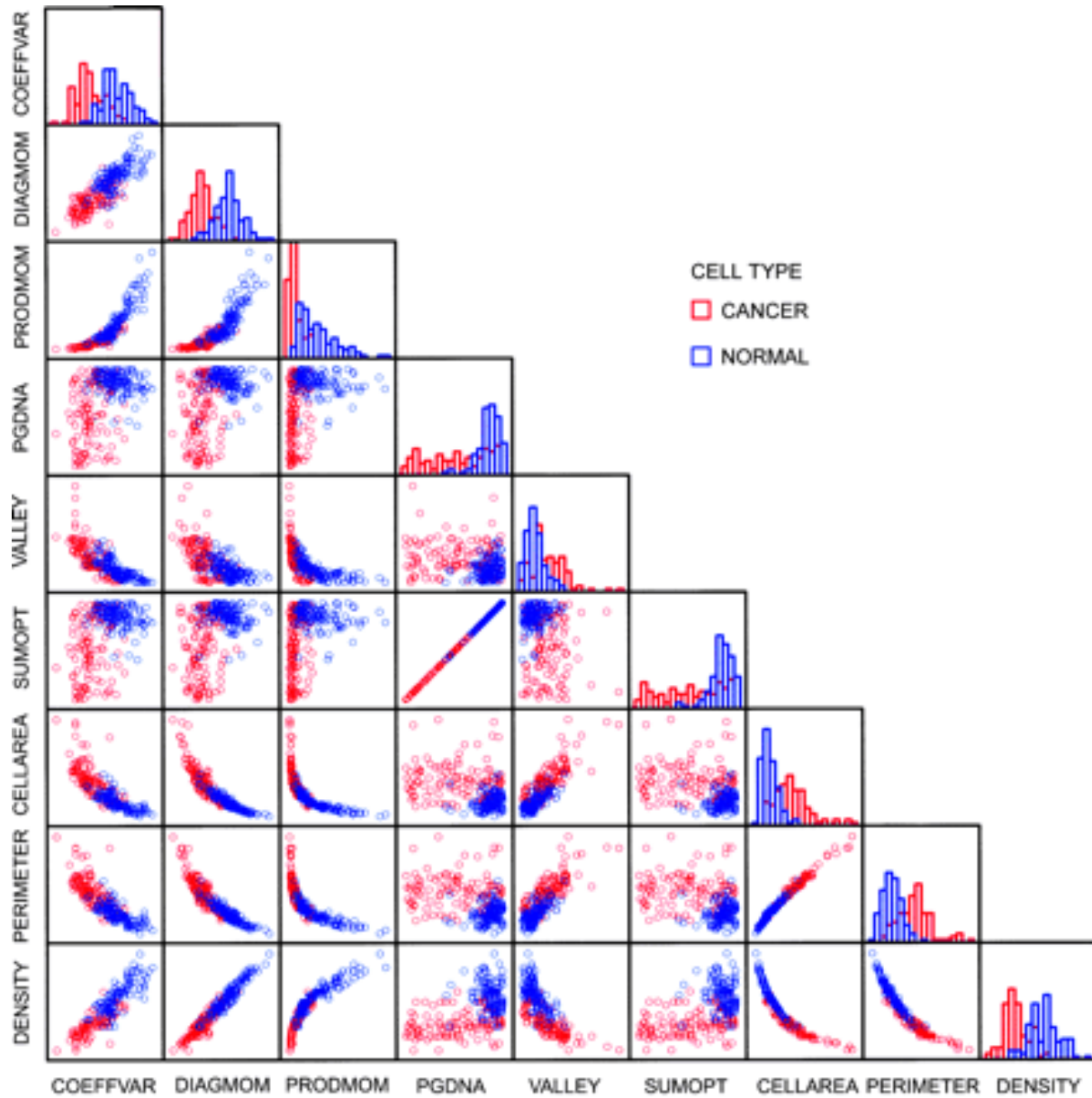


Image Source: Wikipedia



# scatterplot matrix

movie IMDB ID

tt1430132

Load

## The Wolverine

2013 - 2 h 6 min

### Actors

Hugh Jackman (20)

Will Yun Lee (6)

Tao Okamoto (0)

Rila Fukushima (0)

### Directors

James Mangold (6)

### Writers

Mark Bombback (6)

Scott Frank (8)

### Genres

Action (779)

Adventure (563)

Fantasy (366)

Sci-Fi (350)

### Budgets

120000000 (238)

### Producers

Hugh Jackman (2)

Tom Cohen (0)

Stan Lee (27)

Hutch Parker (1)

### Costume\_Designers

### Composers

Marco Beltrami (40)

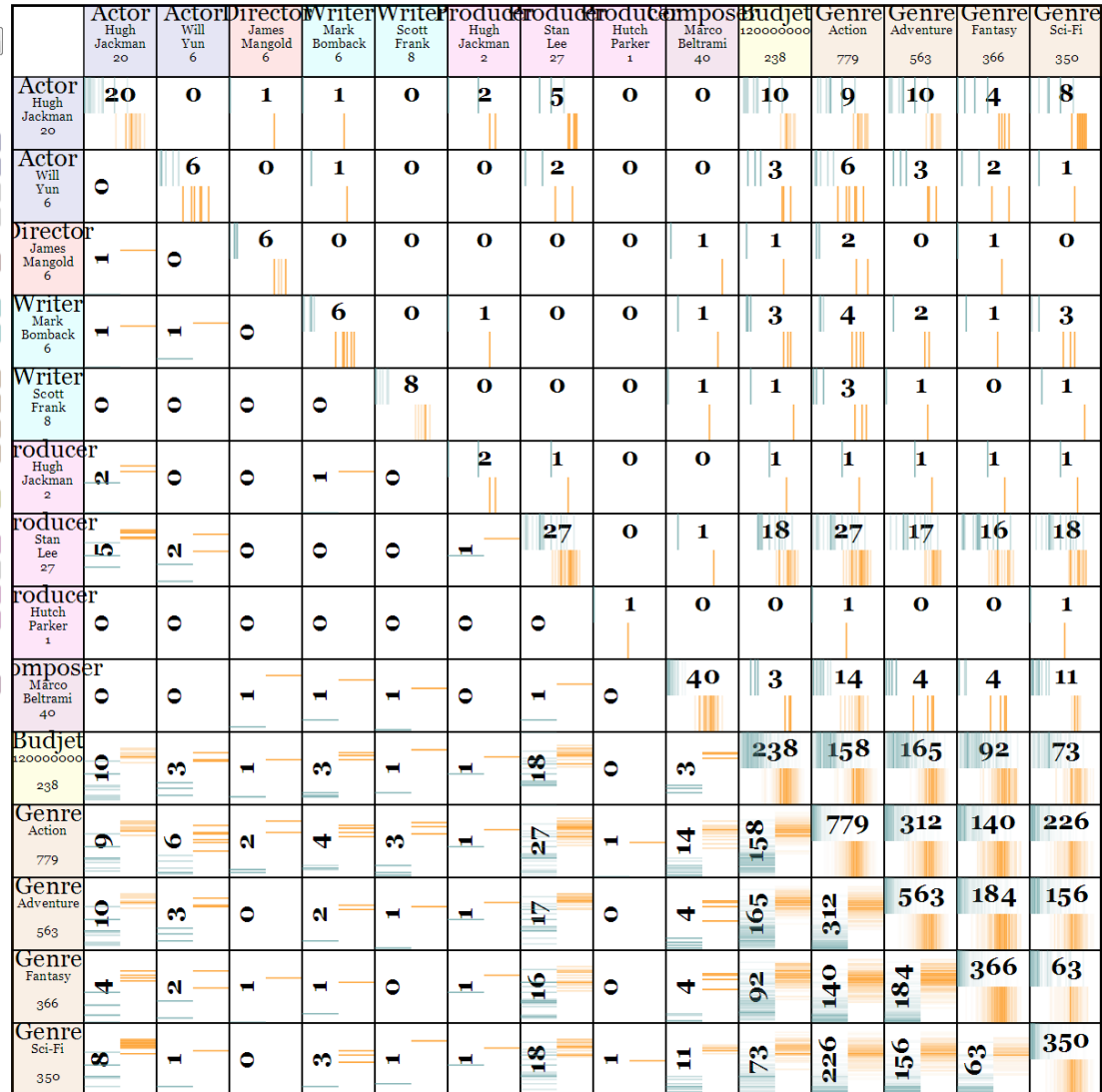
### Cinematographers

### Additional Informations

Composer

Marco Beltrami

Add



By Charles Perin

*ScatterDice*

# Rolling the Dice

Multidimensional Visual Exploration  
using Scatterplot Matrix Navigation

Niklas Elmqvist  
Pierre Dragicevic  
Jean-Daniel Fekete

INRIA

# *small multiple* (sometimes called *trellis chart*, *lattice chart*, *grid chart*, *panel chart*)

- Series of related graphs/charts/plots typically layed out in a grid-structure
- A Scatterplot matrix is a type of Small Multiple – but you can create and use many types of graphs and charts in this manner from your MD datasets

Concept  
introduced by  
Edward Tufte

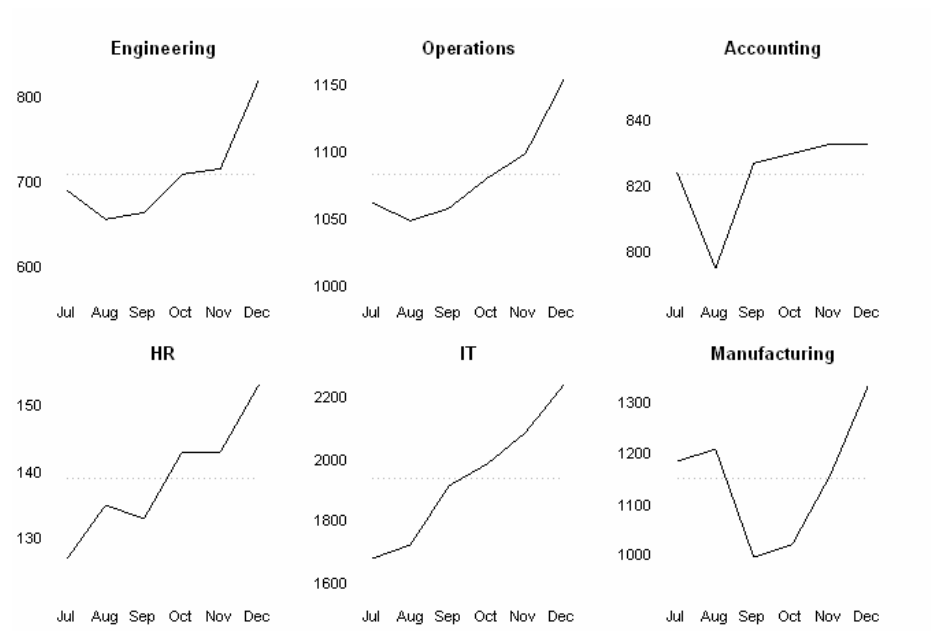


Image Source: Wikipedia

# other examples: sm + maps

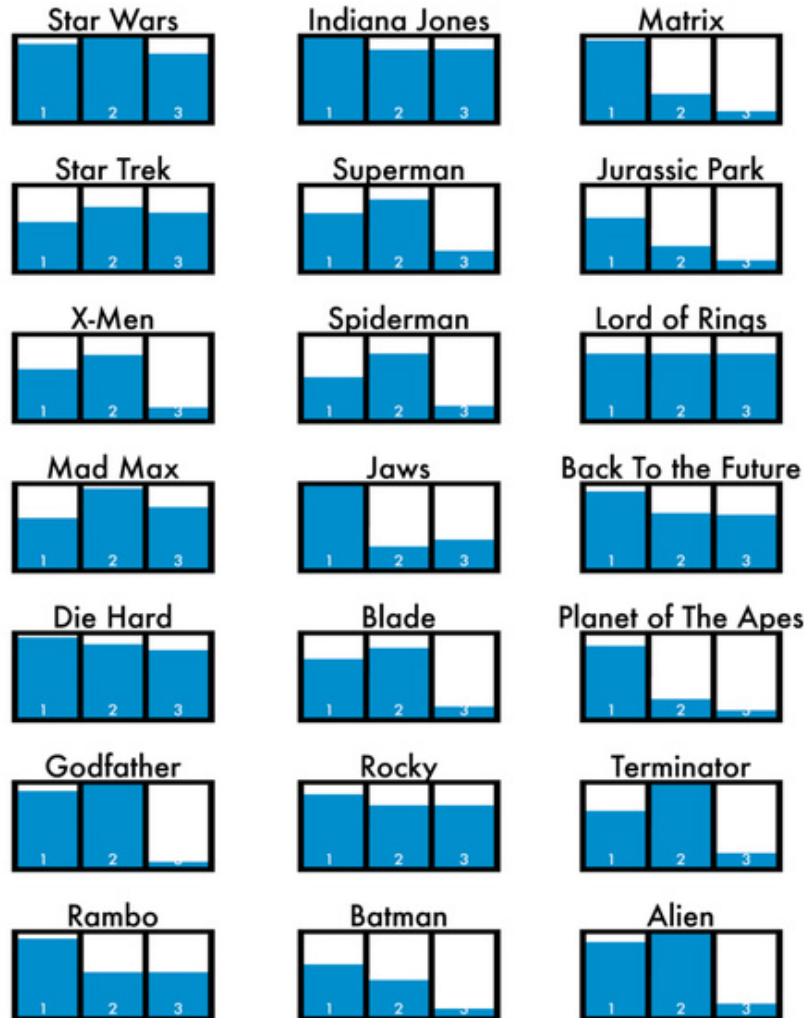
2000: State-level support (orange) or opposition (green) on school vouchers, relative to the national average of 45% support



Orange and green colors correspond to states where support for vouchers was greater or less than the national average. The seven ethnic/religious categories are mutually exclusive. "Evangelicals" includes Mormons as well as born-again Protestants. Where a category represents less than 1% of the voters of a state, the state is left blank.

*other examples: sm + bar charts*

## THE TRILOGY METER



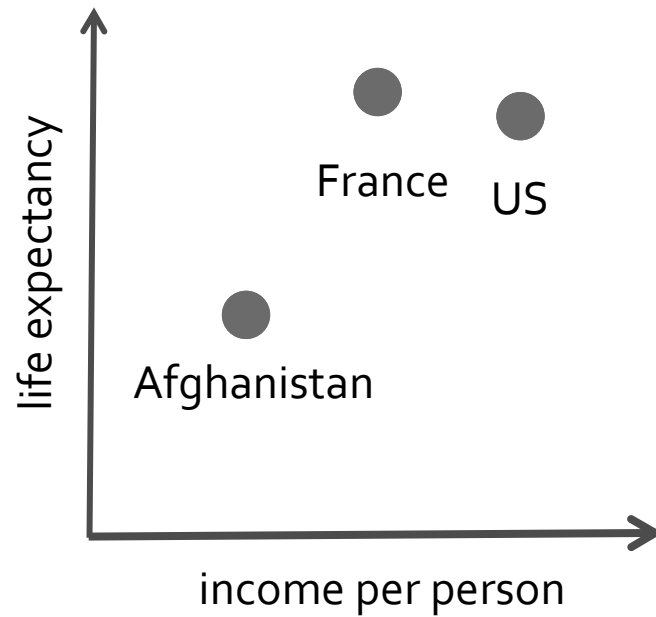
## *design choices to consider for small multiples*

- Order the individual charts logically
- Use appropriate charts for your data
  - use comparable scales
  - use comparable encodings
- Use the same size for all charts & choose an appropriate size

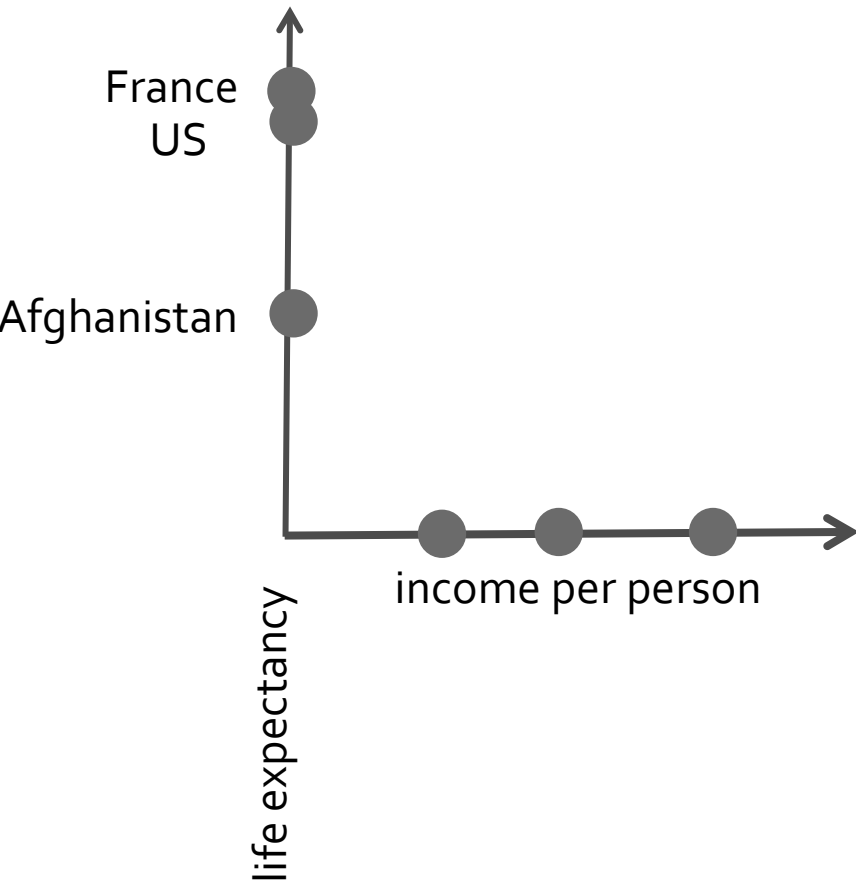


# *Parallel Coordinates*

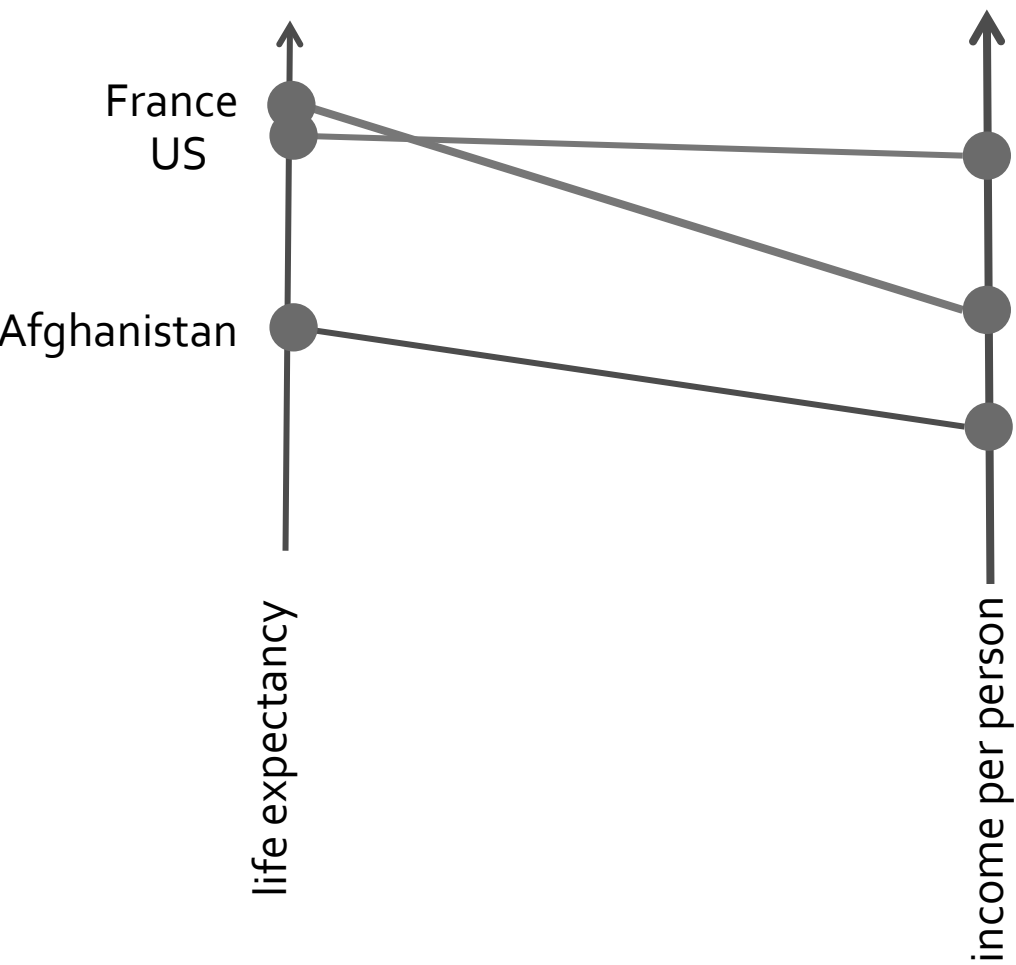
Back to our original example



# *Parallel Coordinates*



# *Parallel Coordinates*



# Parallel Coordinates

## MULTIDIMENSIONAL DETECTIVE

Alfred Inselberg\*, Multidimensional Graphs Ltd†

&

Computer Science Department

Tel Aviv University, Israel

aaisreal@math.tau.ac.il

### Abstract

The display of multivariate datasets in parallel coordinates, transforms the search for *relations* among the variables into a 2-D pattern recognition problem. This is the basis for the application to *Visual Data Mining*. The Knowledge Discovery process together with some general guidelines are illustrated on a dataset from the production of a VLSI chip. The special strength of parallel coordinates is in modeling **relations**. As an example, a simplified Economic Model is constructed with data from various economic sectors of a real country. The visual model shows the interrelationship and dependencies between the sectors, circumstances where there is competition for the same resource, and feasible economic policies. Interactively, the model can be used to do trade-off analyses, discover sensitivities, do approximate optimization, monitor (as in a Process) and Decision Support.

### Introduction

In Geometry parallelism, which does not require a notion of angle, rather than orthogonality is the more fundamental concept. This, together with the fact that orthogonality "uses-up" the plane very

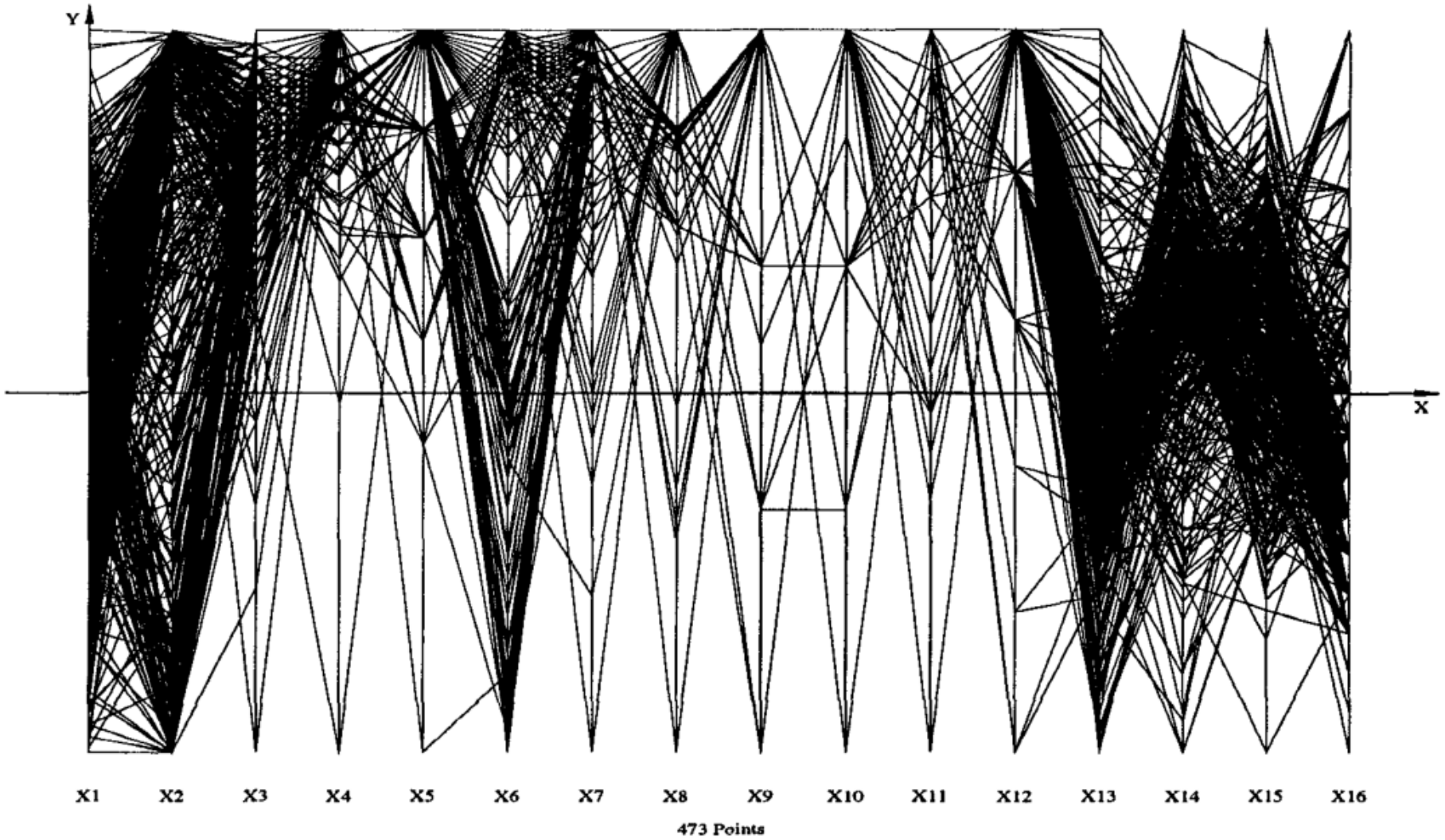
fast, was the inspiration in 1959 for "Parallel" Coordinates. The systematic development began in 1977 [4]. The goals of the program were and still are (see [6] and [5] for short reviews) the visualization of multivariate/multidimensional problems without loss of information and having the properties:

1. Low representational complexity. Since the number of axes,  $N$  equals the number of dimensions (variables) the complexity is  $O(N)$ ,
2. Works for any  $N$ ,
3. Every variable is treated uniformly (unlike "Chernoff Faces" and various types of "glyphs"),
4. The displayed object can be recognized under projective transformations (i.e. rotation, translation, scaling, perspective),
5. The display easily/intuitively conveys information on the properties of the  $N$ -dimensional object it represents,
6. The methodology is based on rigorous mathematical and algorithmic results.

Parallel coordinates (abbr. ||-coords) transform multivariate relations into 2-D patterns, a property that is well suited for Visual Data Mining.

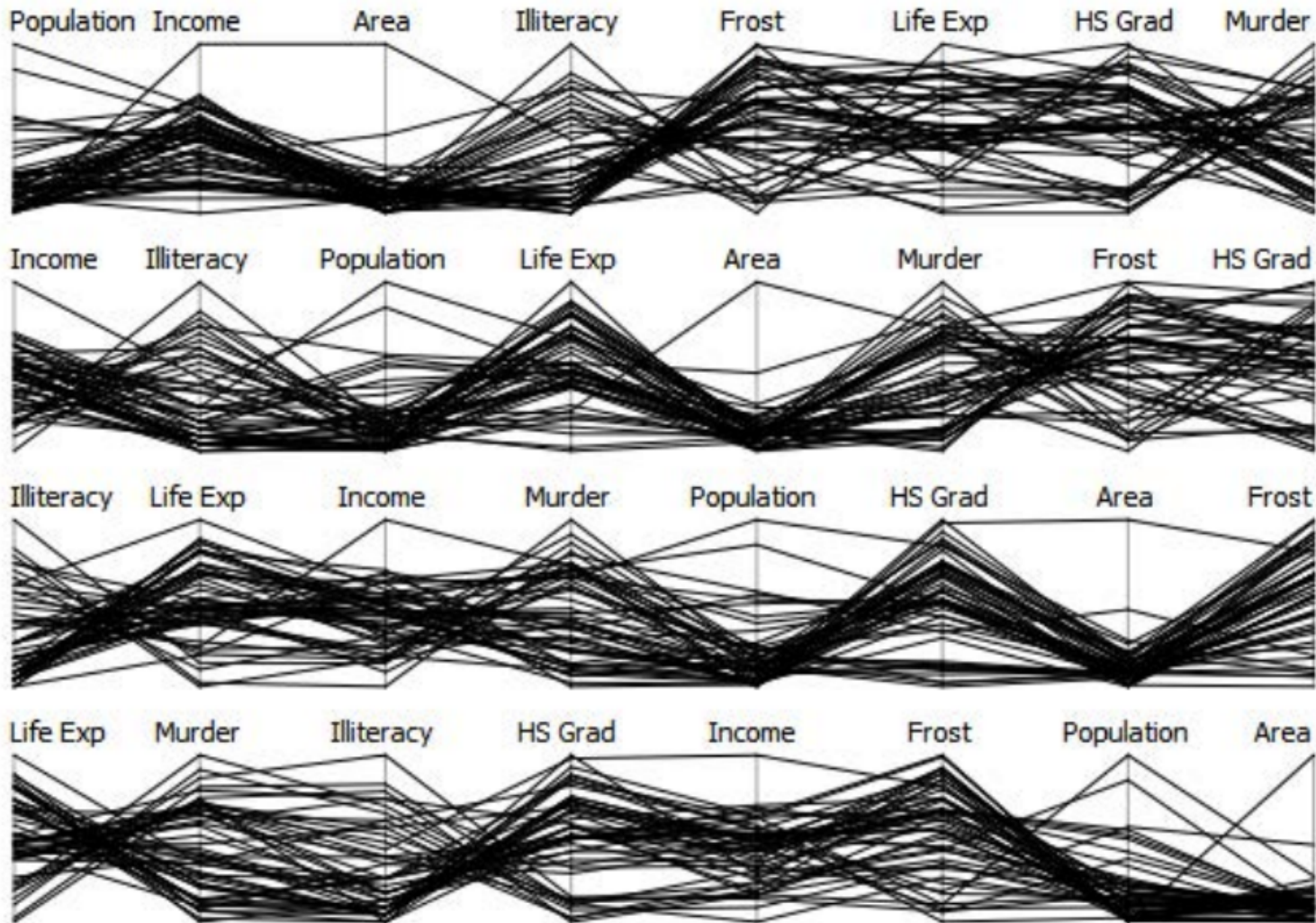
\* Senior Fellow San Diego SuperComputing Center

† 36A Yehuda Halevy Street, Raanana 43556, Israel



Original Example from Inselberg 1997

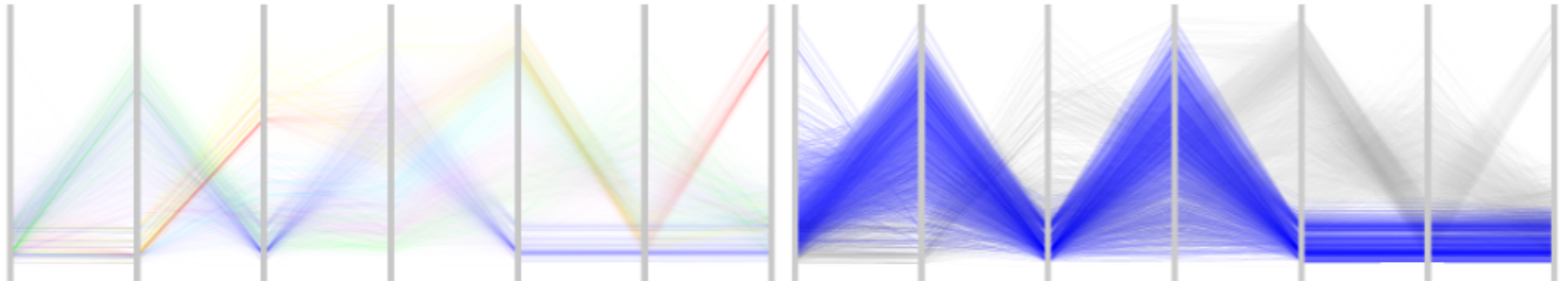
# *the order of axes matters*



Eurographics 2013, STAR Report  
J. Heinrich, D. Weiskopf

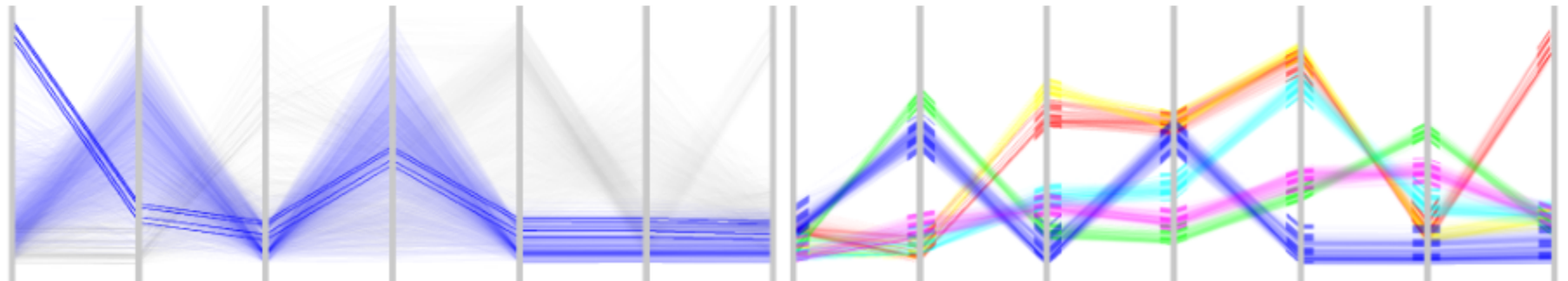
# *reduce clutter - highlight clusters*

Lots of work on this. For example:



(a) A linear transfer function has been applied to the high-precision texture in order to prevent cluttering and to provide overview of the data.

(b) A logarithmic transfer function is applied to a selected cluster. The structure is preserved and emphasis is put on the low density regions.



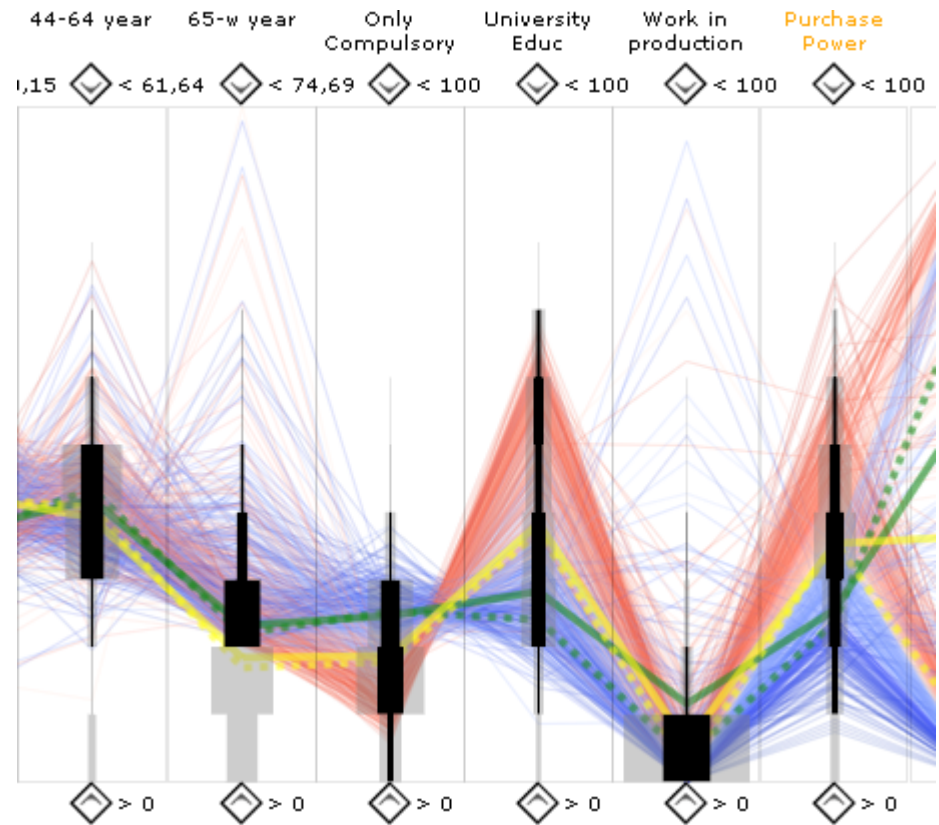
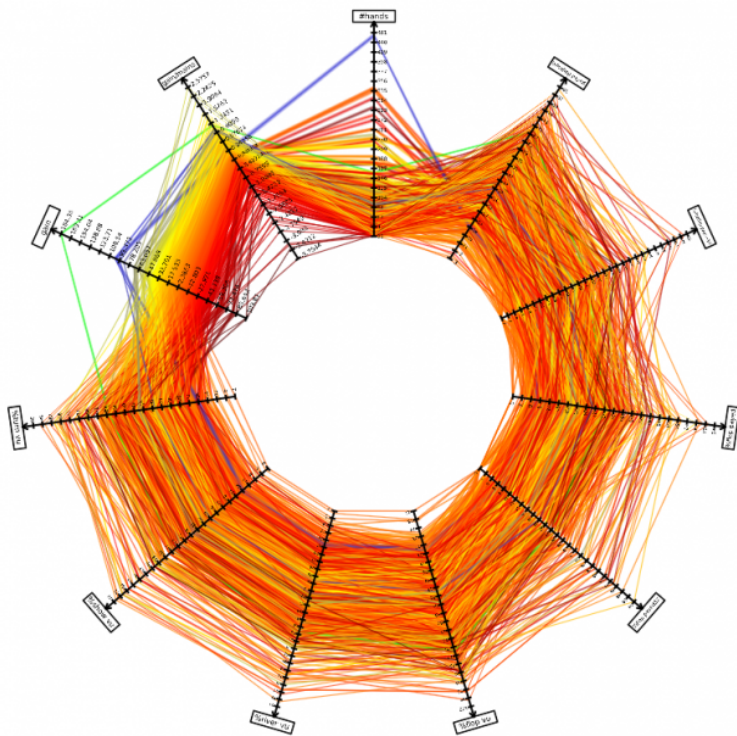
(c) Local cluster outliers are enhanced. A square root transfer function is used and the outliers are visible even through high-density regions.

(d) A complementary view of the clusters with uniform bands. 'Feature animation' presents statistics about the clusters and acts as a guidance.

Revealing Structure within Clustered Parallel Coordinates Displays, InfoVis 2005

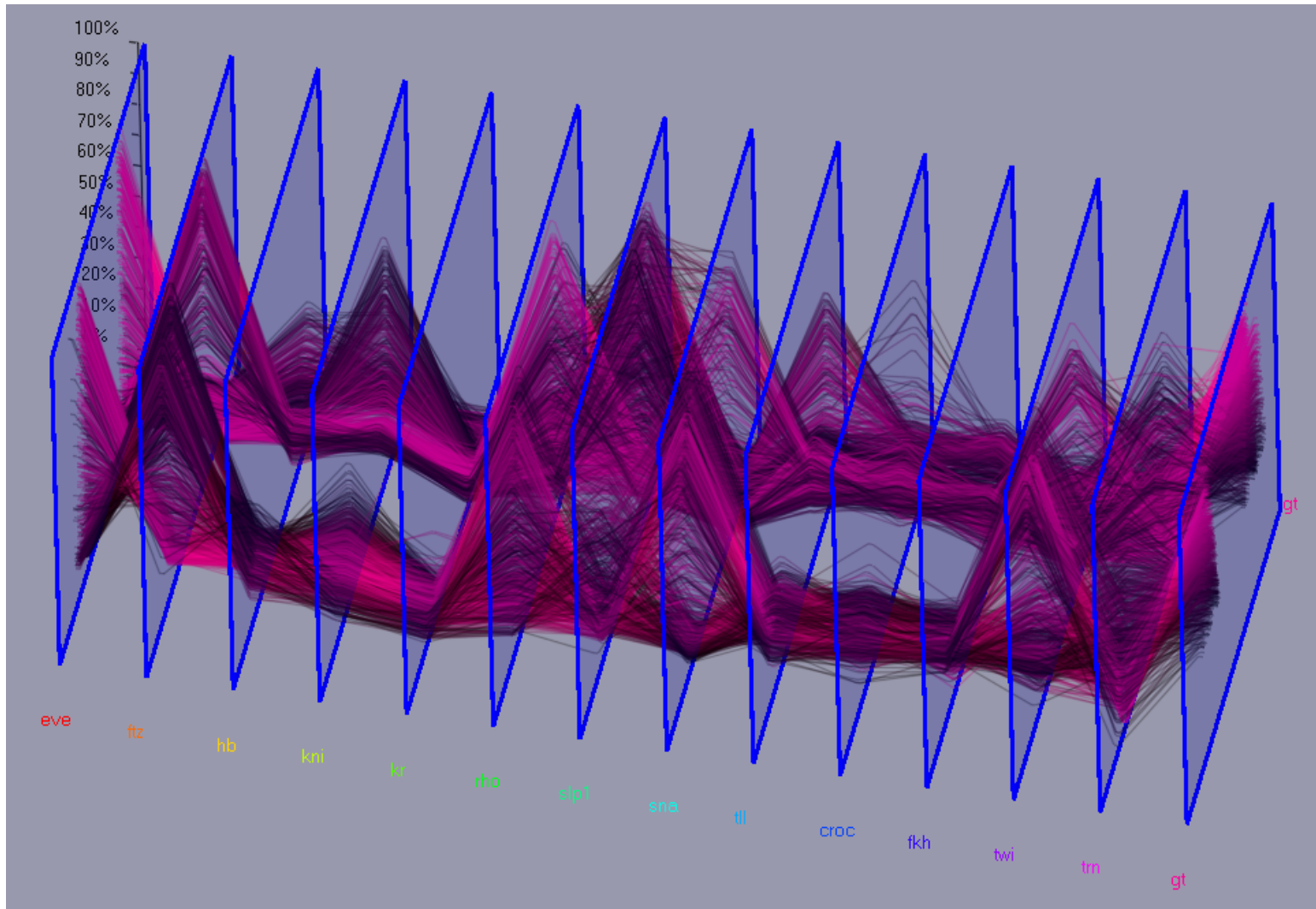
# Other variants

- Other layout, tulip (circular)
- PC with summary stats



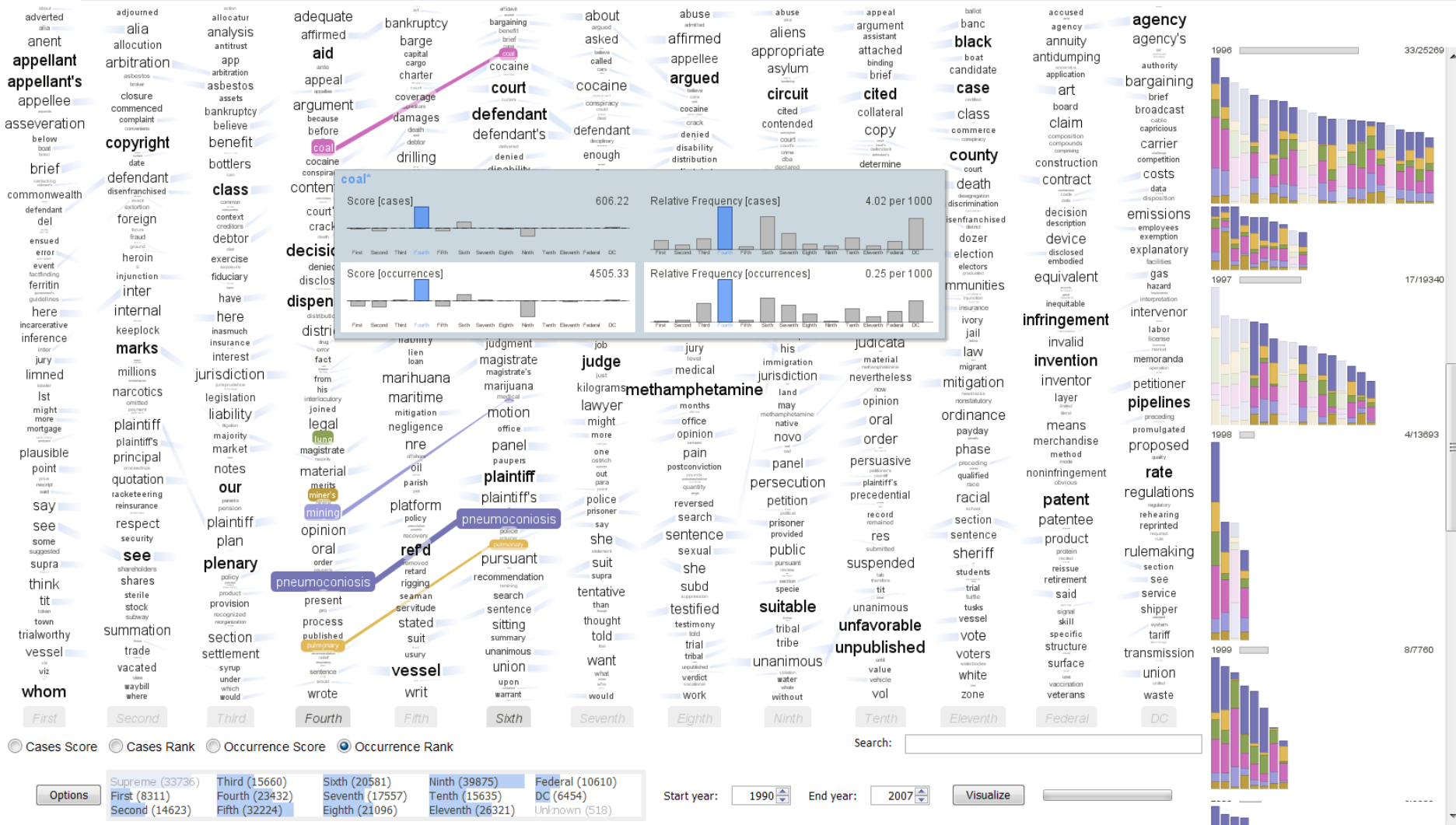


# 3D Parallel Coordinate



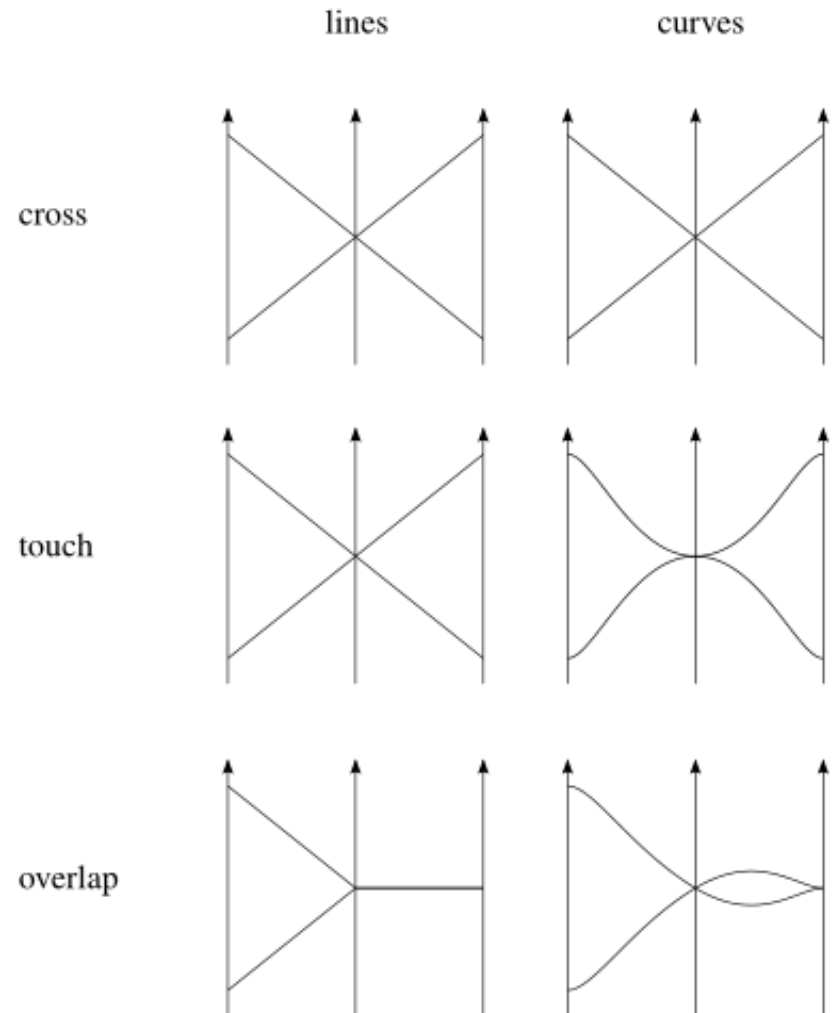
# combine with other visualization techniques

## Parallel Tag Clouds to Explore Faceted Text Corpora (Collins et al., VAST 2009)



# *how to draw the lines*

Goal: avoid ambiguity



*star glyphs + parallel coordinates*

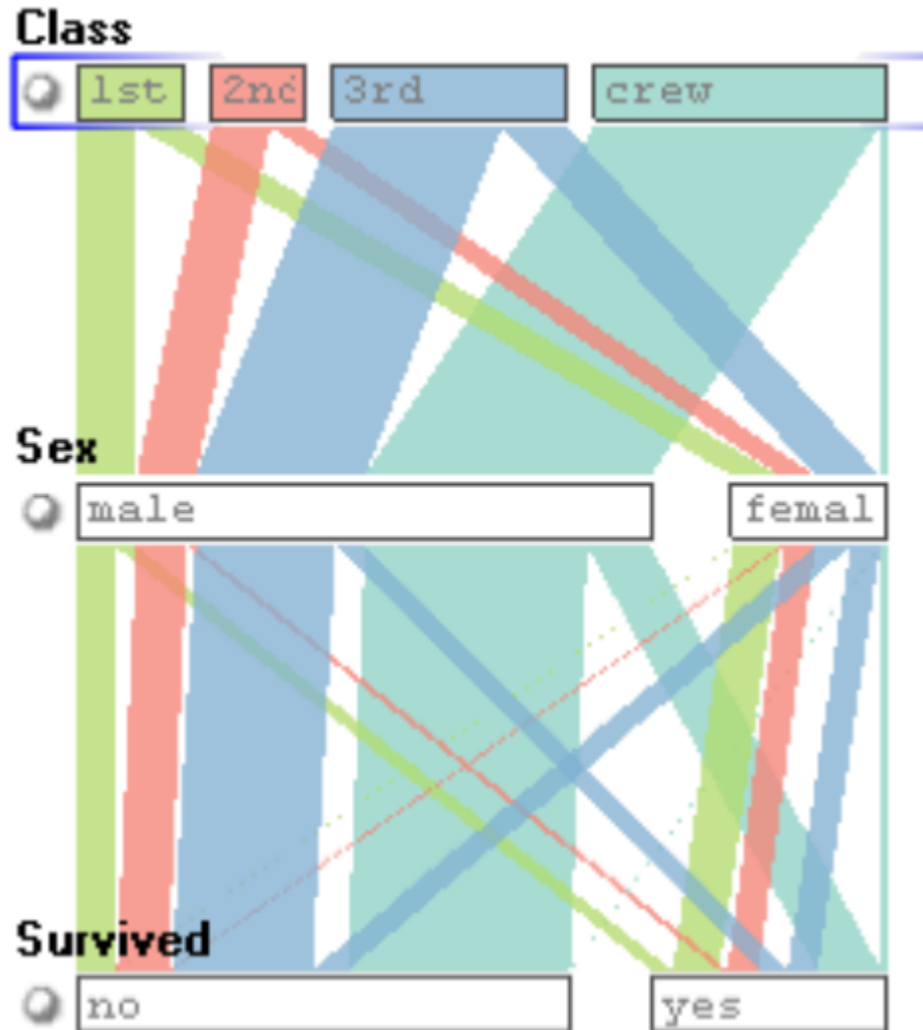
# **Parallel Glyphs**

**Elena Fanea**

**Sheelagh Carpendale**

**Tobias Isenberg**

# Parallel sets – categorical data



# there is much more on this...

- Start here if you want more information

EUROGRAPHICS 2013/M. Sbert, L. Szirmay-Kalos

STAR – State of The Art Report

## State of the Art of Parallel Coordinates

J. Heinrich and D. Weiskopf

Visualization Research Center, University of Stuttgart

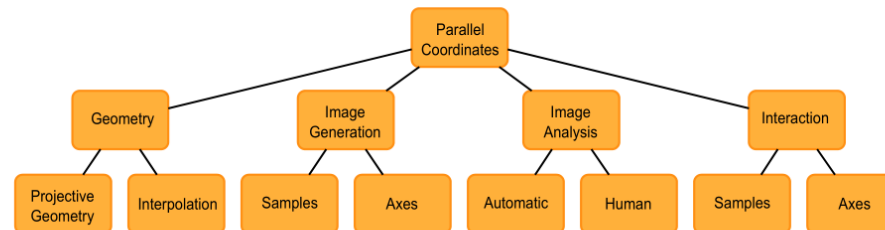


Figure 1: Taxonomy of topics for parallel coordinates in the scientific literature. The first-level nodes each represent a section in this paper, where the scope and definition of each topic will be explained.

### Abstract

*This work presents a survey of the current state of the art of visualization techniques for parallel coordinates. It covers geometric models for constructing parallel coordinates and reviews methods for creating and understanding visual representations of parallel coordinates. The classification of these methods is based on a taxonomy that was established from the literature and is aimed at guiding researchers to find existing techniques and identifying white spots that require further research. The techniques covered in this survey are further related to an established taxonomy of knowledge-discovery tasks to support users of parallel coordinates in choosing a technique for their problem at hand. Finally, we discuss the challenges in constructing and understanding parallel-coordinates plots and provide some examples from different application domains.*

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Picture/Image Generation—Line and curve generation

# *Paralle Coordinates*

- Try it out
  - XmdvTool <http://davis.wpi.edu/%7Exmdv/index.html>
  - Parvis <http://www.mediavirus.org/parvis/>
  - Macrofocus <http://www.macrofocus.com/public/products/infoscope.html>

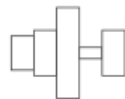
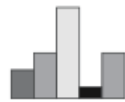
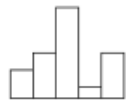
# Classification of MD Vis Techniques

Icon-based



# glyphs

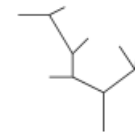
- Small composite visual representations of multi-dimensional data points
- Often used in small multiple setting
- Characterized generally by lack of reference structures (grid lines, axes labels, ...)



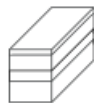
Variations on Profile glyphs



Stars and Anderson/metroglyphs



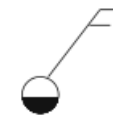
Sticks and Trees



Autoglyph and box glyph



Face glyphs



Arrows and Weathervanes

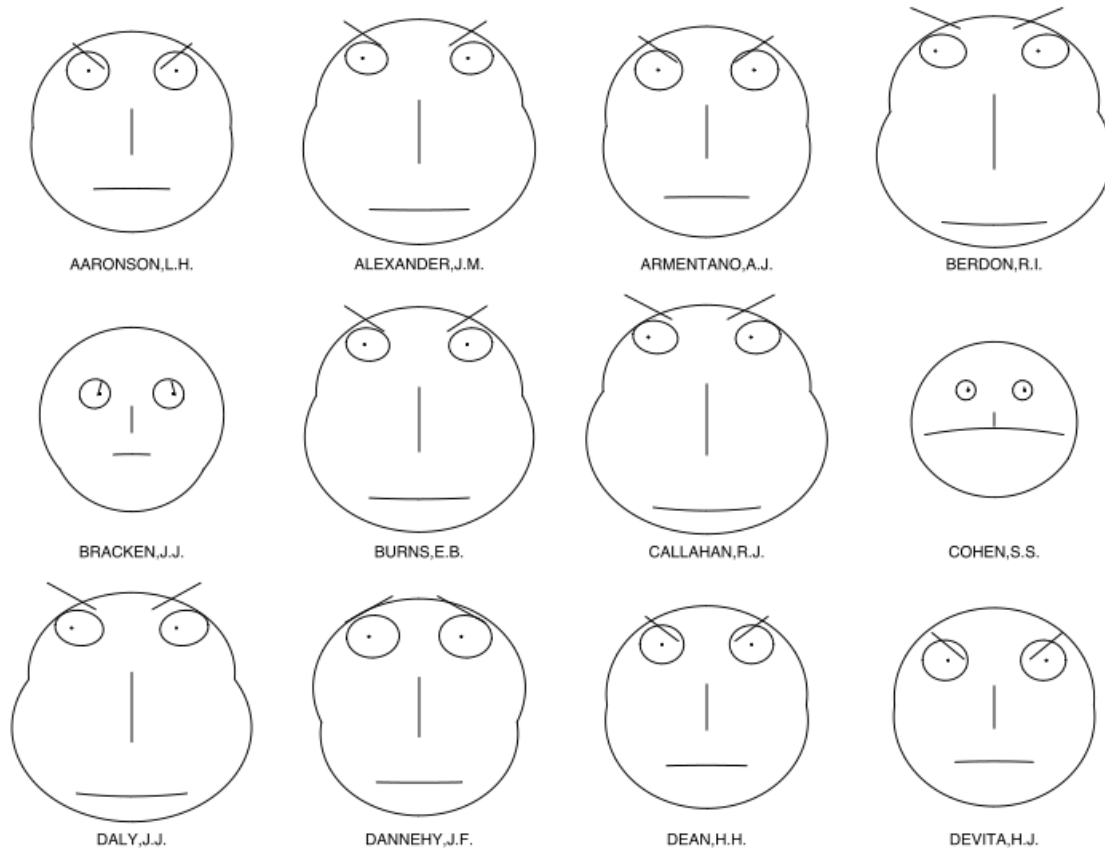
From Ward, 2002

A taxonomy of glyph placement strategies for multidimensional data visualization

## *mappings from dimension to visual encoding*

- One-to-one:
  - each dimension maps to a distinct and different visual encoding

# *example: chernoff faces*



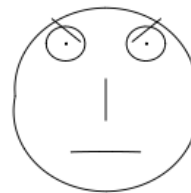
Herman Chernoff,  
[The Use of Faces to Represent Points in K-Dimensional  
Space Graphically](#), 1973.

# *example: chernoff faces*

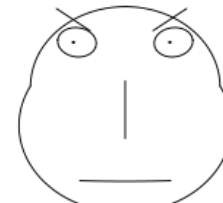
- features of a human face encode data values

- 10 Parameters:

- Head Eccentricity
- Eye Eccentricity
- Pupil Size
- Eyebrow Slope
- Nose Size
- Mouth Vertical Offset
- Eye Spacing
- Eye Size
- Mouth Width
- Mouth Openness



AARONSON, L.H.



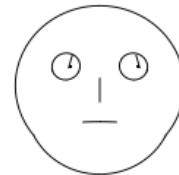
ALEXANDER, J.M.



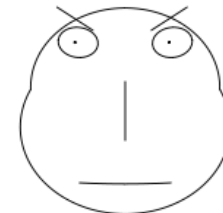
ARMENTANO, A.J.



BERDON, R.I.



BRACKEN, J.J.



BURNS, E.B.



CALLAHAN, R.J.



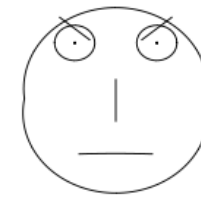
COHEN, S.S.



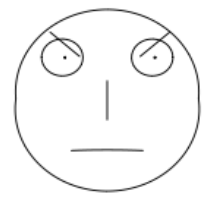
DALY, J.J.



DANNEHY, J.F.



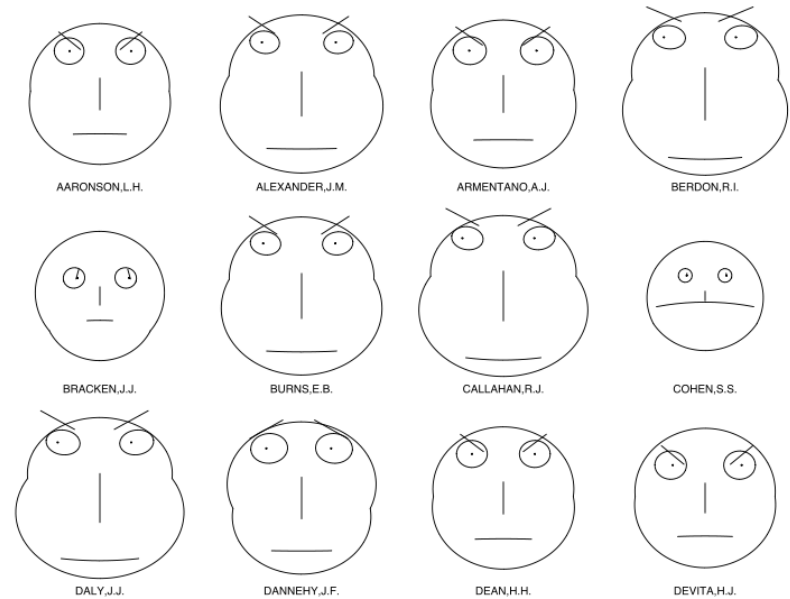
DEAN, H.H.



DEVITA, H.J.

# *chernoff faces*

- reasoning: humans are good at differentiating faces and reading face features
- problem: chernoff faces have generally found not to be very effective

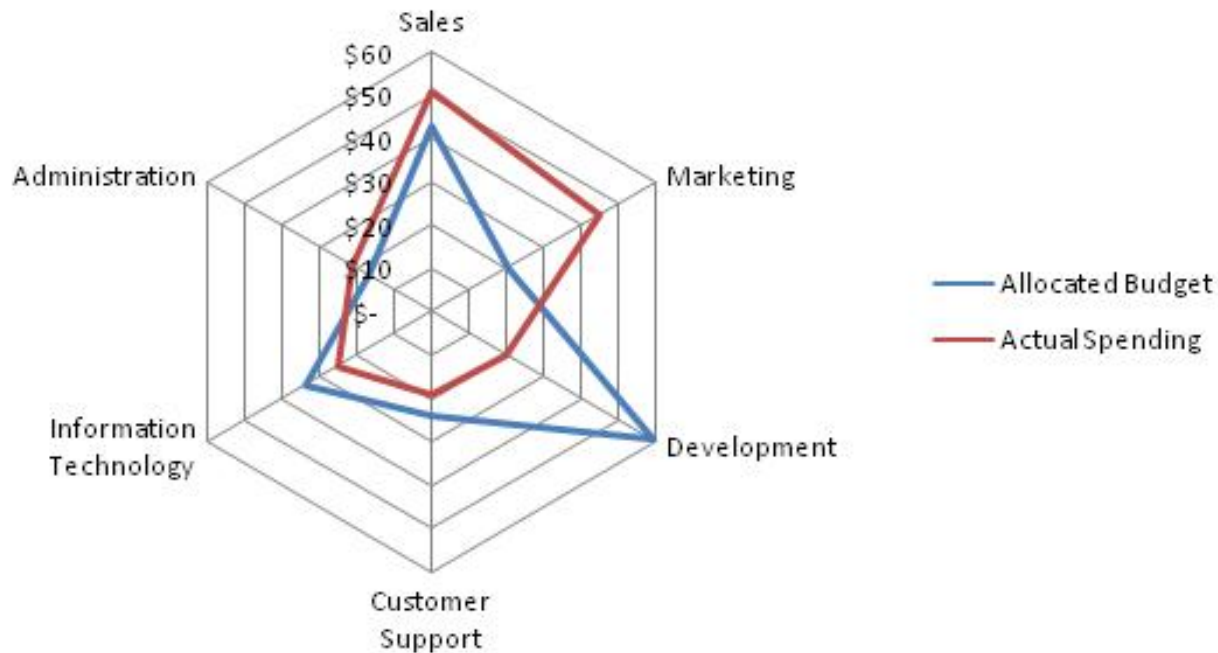


# *mappings from dimension to visual encoding*

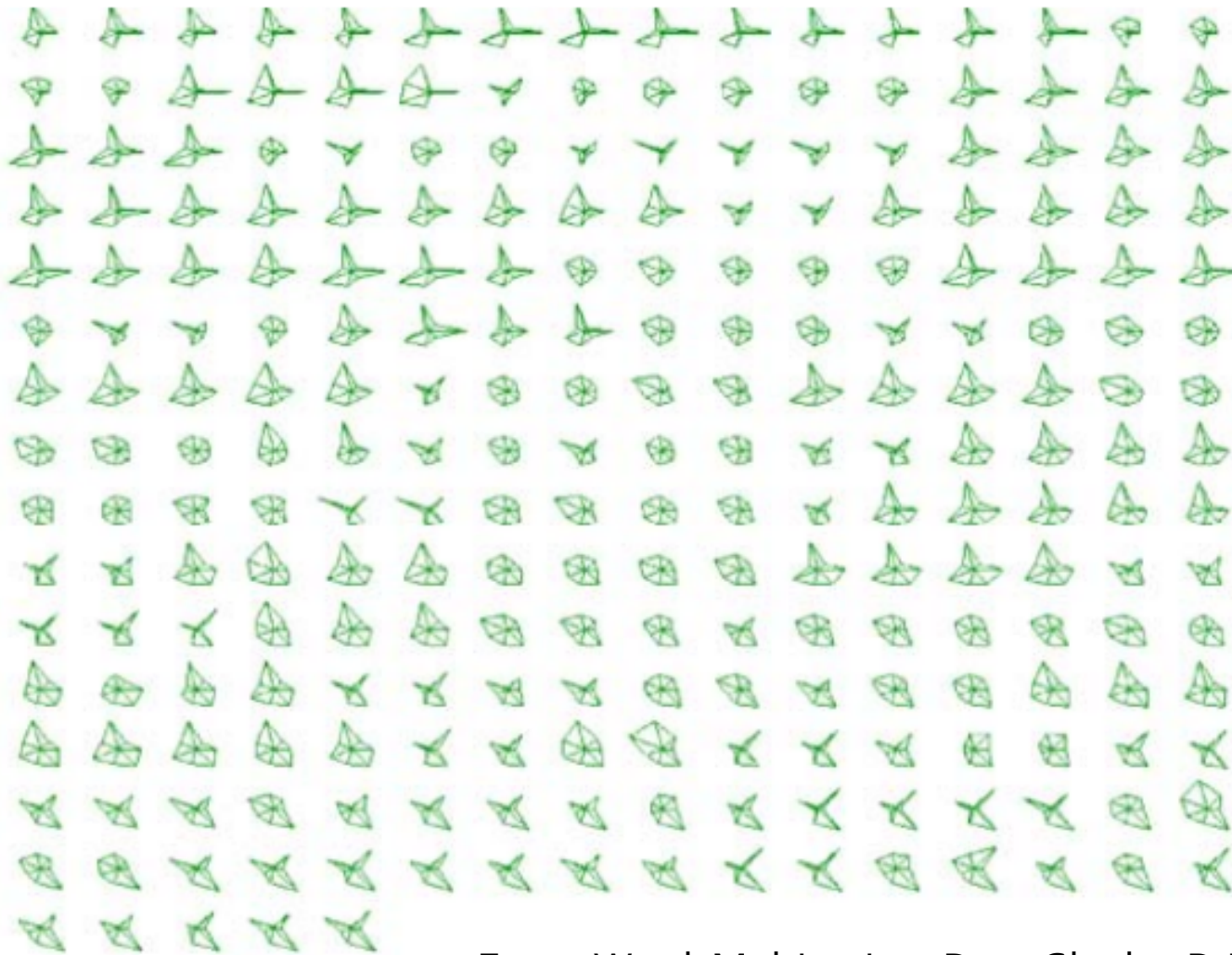
- One-to-one:
  - each dimension maps to a distinct and different visual encoding
- One-to-many
  - one dimension encoded in more than one way
  - used to improve accuracy & ease of interpreting the data (e.g. for color blind viewers)
- Many-to-one
  - several or all dimensions use the same visual encoding, separated in space, orientation, or otherwise

# *star glyphs*

- Lay out dimension in radial fashion
- Draw each point as a closed polyline



# *star glyphs*

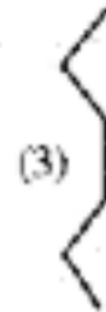
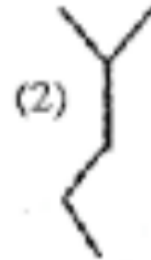
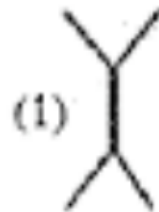
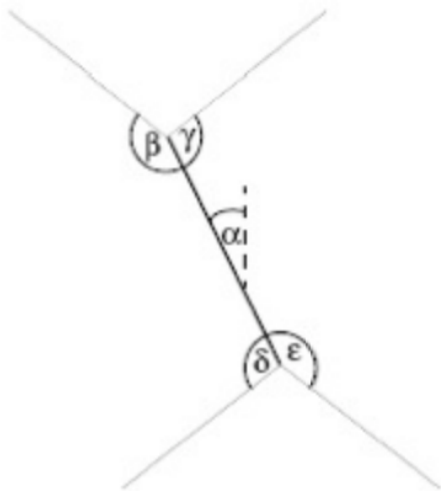


From: Ward Multivariate Data Glyphs: Principles and Practice. Handbook of Data Visualization (2008)

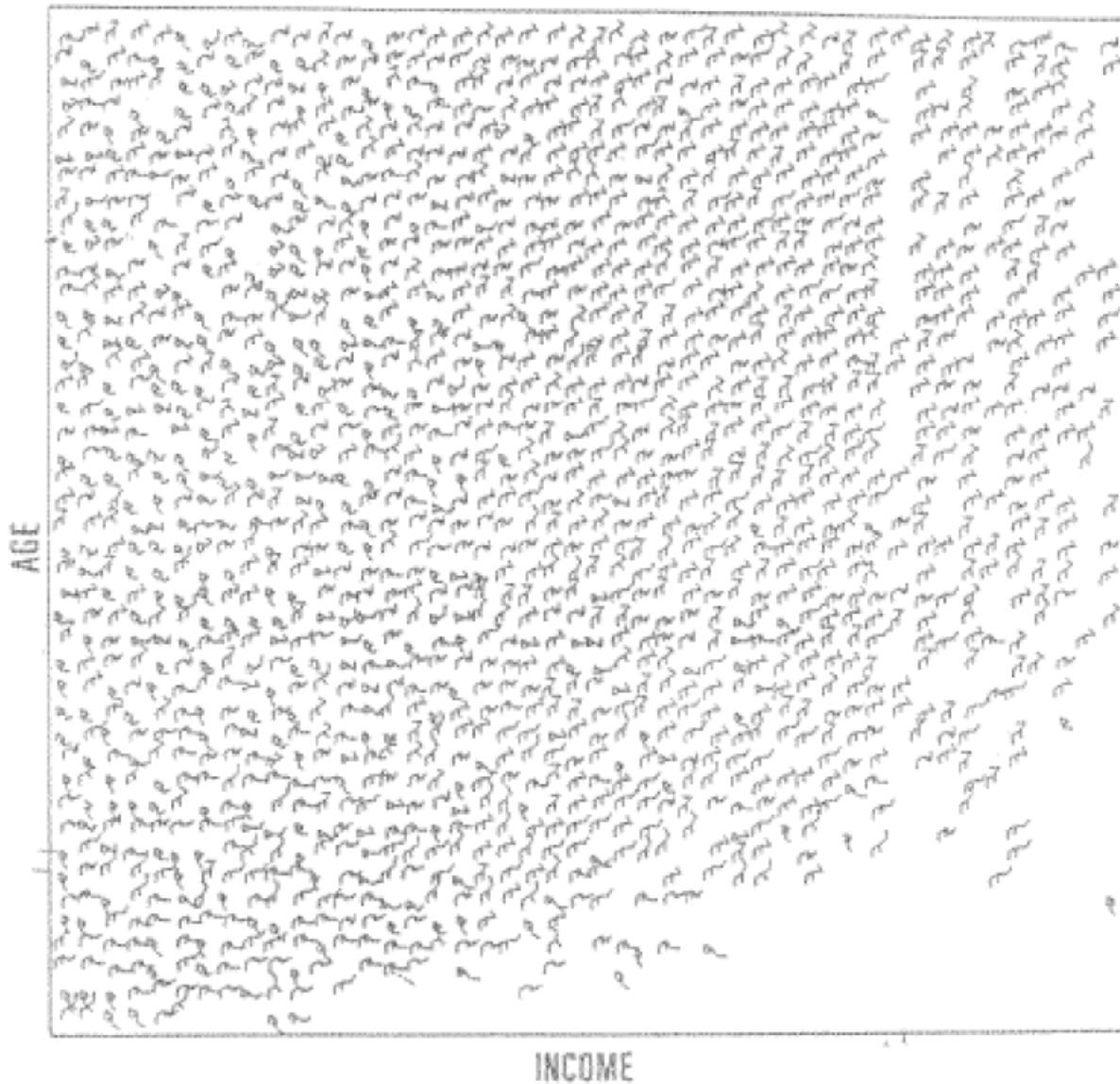


# Stick figure Icons (Pickett & Grinstein)

- two attributes are mapped to the display axes
- remaining attributes are mapped to lengths of limbs or angles between them
- Idea: Texture pattern in visualization shows certain characteristics.



# Stick figure Icons (Pickett & Grinstein)



Census data showing age (y), income (x), education, salary, language, marital status etc

# *Glyph-Based MD Visualization*

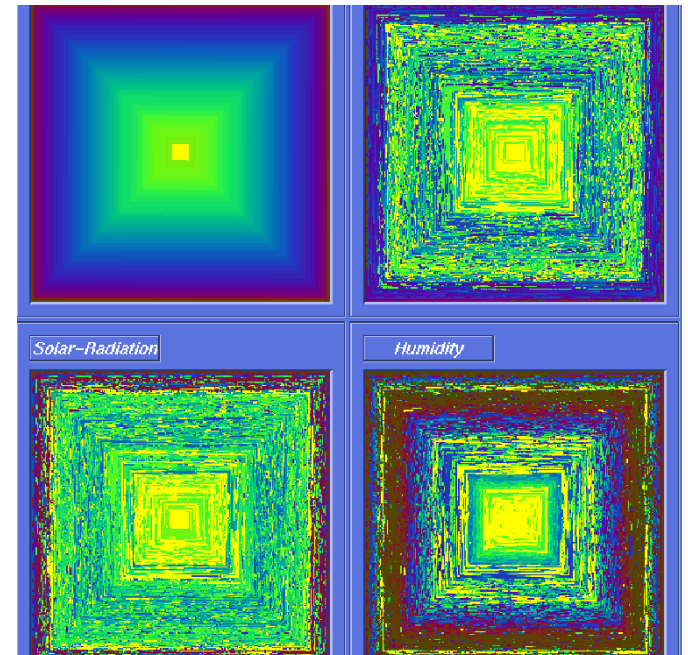
- Pros
  - Provide holistic overview of the information space
  - Exploit the human powerful ability of perceiving (texture)
    - patterns and human face characteristics (Chernoff)
  - Direct metaphor of Chernoff-face-like icons (e.g. houses)
  - may prove to be intuitive for novice users
- Cons
  - Glyphs must be learned
  - Only suitable for small to medium data sets
  - stick figures give a rather broad overview and may be difficult to interpret
  - Mappings may introduce biases in interpretation (e.g. the head shape of a Chernoff-face may be easier to perceive and compare than length of nose)

# Classification of MD Vis Techniques

## Pixel-oriented Techniques

# VisDB

- Database of data items, each of  $n$  dimensions
- Issue a query that specifies a target value of the dimensions
- Often get back no exact matches
- Want to find near matches
- Relevance factor
- metadata



Taken from:  
D. Keim, H-P Kriegel, "VisDB Database Exploration  
Using Multid Vis", IEEE CG&A, 1994.

# *Technique*

- Calculate relevance of all data points
- Sort items based on relevance
- Use spiral technique to order the values
- Color items based on relevance



High

Empirically established

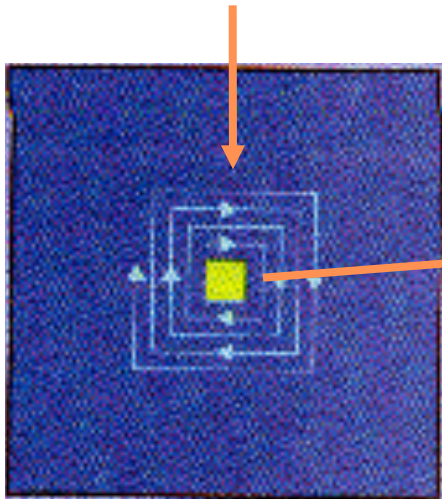
Low

# Display Methodology

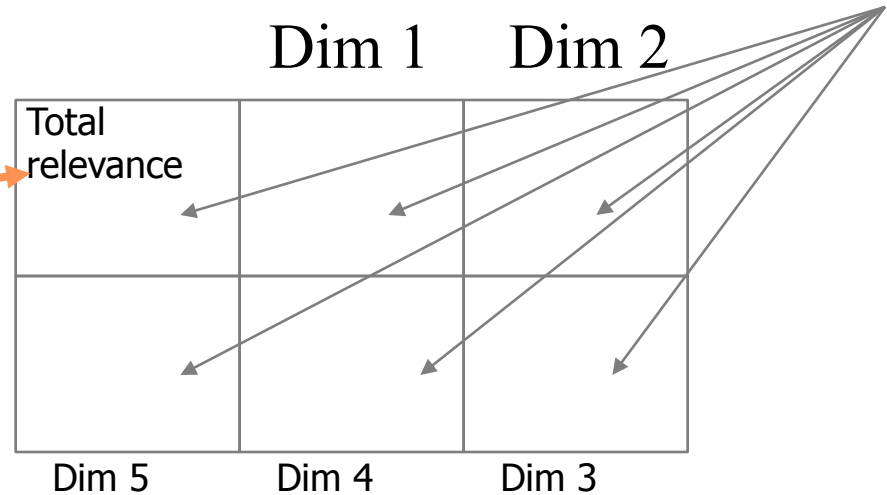
Items ordered by total relevance

Highest relevance value in center, decreasing values grow outward

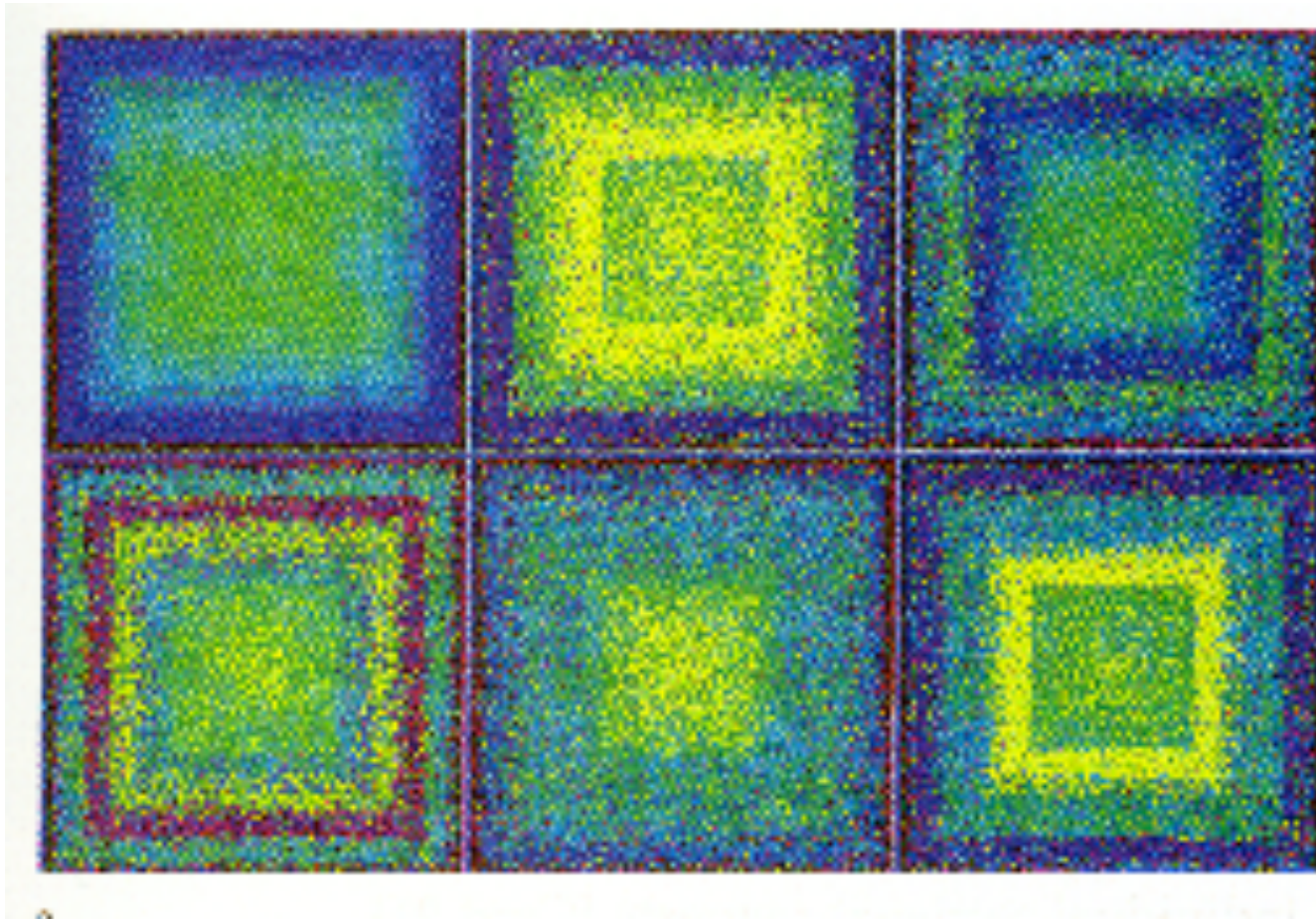
Same item appears in same place in each window



Spiral in each window



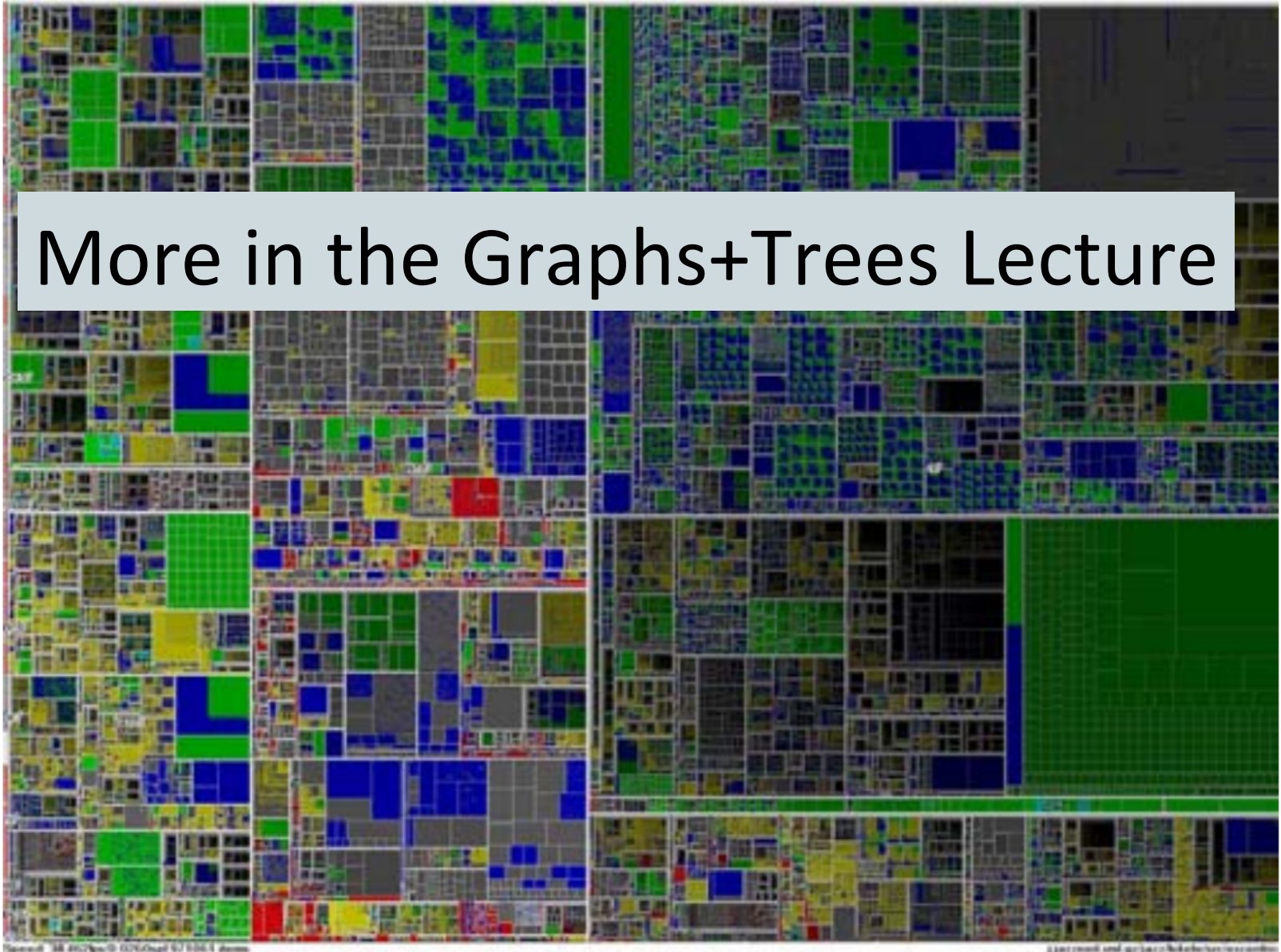
# *VisDB Example*





# *Directory Structure using treemap ...*

More in the Graphs+Trees Lecture



## *Back to: two main approaches*

1. Attempt to visualize all dimensions
2. **Visualize only parts of the data**  
→ interaction

## *Back to: two main approaches*

1. Attempt to visualize all dimensions
2. **Visualize only parts of the data**
  - interaction
  - Dimension Reduction

## *Back to: two main approaches*

1. Attempt to visualize all dimensions
2. **Visualize only parts of the data**
  - interaction
  - Dimension Reduction

**which dimensions to choose?**

# *Dimension Reduction - Approaches*

- Variable / feature selection
- Combined dimensions: PCA, MDS
- Dimension clustering

# *Feature selection*

- the process of selecting a subset of relevant features
- redundant features are those which provide no more information than the currently selected features
  - simple to perform
  - intuitive to understand
  - continuous process

# PCA

- *Statistical procedure that uses orthogonal transformations*
- *It 'discovers' new variables, called 'Principle Components' PCs which account for the majority of the **variability** in the data.*
- Enables description of information with considerably fewer variables than original data set.

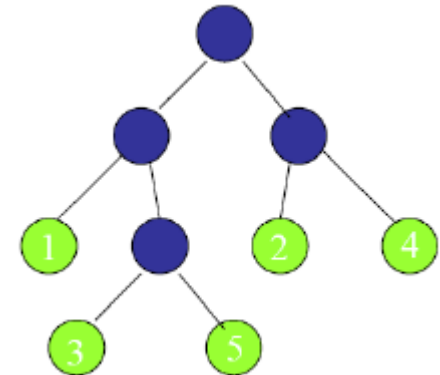
# MDS

- *Multidimensional scaling (MDS) maps the distances between items in a high dimensional space into a lower (e.g., two) dimensional space.*
- Relocate  $n$  items in a 2D dimensional space as points so that the differences between pairs of points in this reduced space match the ordered differences between the subjects in original dimension
- Matching distances: minimize the difference of distances in original and projection data space



# *Hierarchical Dimension Reduction*

- 5-D dataset
- Similar dimensions form clusters
- Clusters are grouped into larger clusters in a dimension hierarchy



# summary

- there are many different kinds of md data
  - today we covered mostly general md-quantitative data
  - MANY more techniques exist that also mix types of data
  - many techniques are designed to solve a domain-specific problem or to help domain experts
  - almost all remaining lectures will also deal with md data

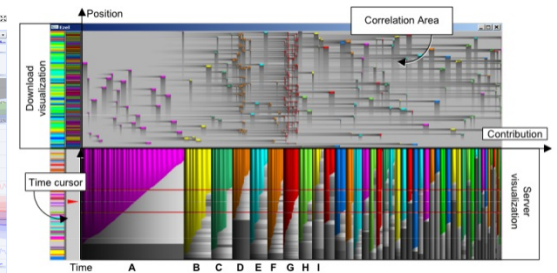
some md examples from lecture 1



graphs



time



text

# *general md-data vis strategies*

- Avoid over-encoding, reduce the problem space
  - ask yourself or your users what needs to be seen
  - ask what we can learn from the data you have, focus on this aspect
  - ask yourself what questions you want your viewers to answer
- Use space and small multiples intelligently
- User interaction to generate relevant views
- Invest time in **interaction** design (see lecture)

*today you...*

- learned about techniques for MD data visualization
- learned relationships between MD visualization techniques
- understood the problems of MD data visualization

# *Acknowledgements*

Slides adapted or inspired from slides authored by:

- Petra Isenberg
- Fred Vernier
- Chris North
- Ye Zhao
- Penny Rheingans
- Remco Cheng